

## Key Ideas

- You can analyse absolute value functions in several ways:
  - graphically, by sketching and identifying the characteristics of the graph, including the  $x$ -intercepts and the  $y$ -intercept, the minimum values, the domain, and the range
  - algebraically, by rewriting the function as a piecewise function
- In general, you can express the absolute value function  $y = |f(x)|$  as the piecewise function

$$y = \begin{cases} f(x), & \text{if } f(x) \geq 0 \\ -f(x), & \text{if } f(x) < 0 \end{cases}$$

- The domain of an absolute value function  $y = |f(x)|$  is the same as the domain of the function  $y = f(x)$ .
- The range of an absolute value function  $y = |f(x)|$  depends on the range of the function  $y = f(x)$ . For the absolute value of a linear or quadratic function, the range will generally, but not always, be  $\{y \mid y \geq 0, y \in \mathbb{R}\}$ .

## Check Your Understanding

### Practise

1. Given the table of values for  $y = f(x)$ , create a table of values for  $y = |f(x)|$ .

a)

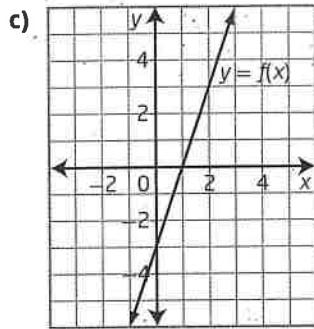
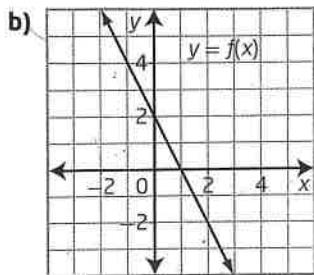
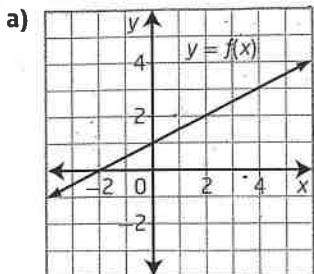
$x$	$y = f(x)$
-2	-3
-1	-1
0	1
1	3
2	5

b)

$x$	$y = f(x)$
-2	0
-1	-2
0	-2
1	0
2	4

2. The point  $(-5, -8)$  is on the graph of  $y = f(x)$ . Identify the corresponding point on the graph of  $y = |f(x)|$ .
3. The graph of  $y = f(x)$  has an  $x$ -intercept of 3 and a  $y$ -intercept of  $-4$ . What are the  $x$ -intercept and the  $y$ -intercept of the graph of  $y = |f(x)|$ ?
4. The graph of  $y = f(x)$  has  $x$ -intercepts of  $-2$  and  $7$ , and a  $y$ -intercept of  $-\frac{3}{2}$ . State the  $x$ -intercepts and the  $y$ -intercept of the graph of  $y = |f(x)|$ .

5. Copy the graph of  $y = f(x)$ . On the same set of axes, sketch the graph of  $y = |f(x)|$ .



6. Sketch the graph of each absolute value function. State the intercepts and the domain and range.

a)  $y = |2x - 6|$

b)  $y = |x + 5|$

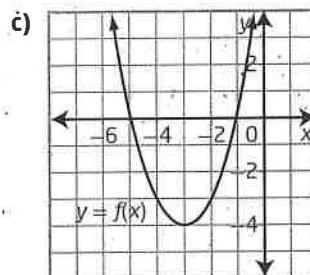
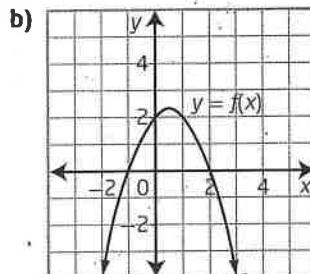
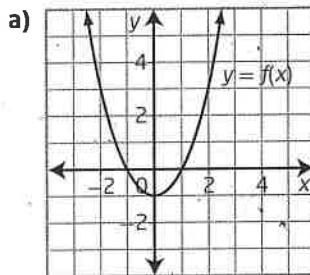
c)  $f(x) = |-3x - 6|$

d)  $g(x) = |-x - 3|$

e)  $y = \left| \frac{1}{2}x - 2 \right|$

f)  $h(x) = \left| \frac{1}{3}x + 3 \right|$

7. Copy the graph of  $y = f(x)$ . On the same set of axes, sketch the graph of  $y = |f(x)|$ .



8. Sketch the graph of each function. State the intercepts and the domain and range.

a)  $y = |x^2 - 4|$

b)  $y = |x^2 + 5x + 6|$

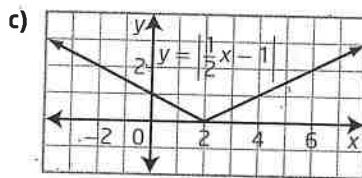
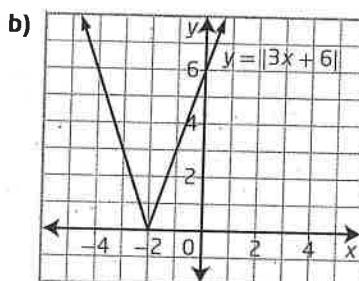
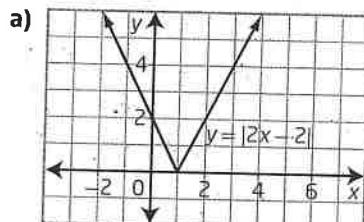
c)  $f(x) = |-2x^2 - 3x + 2|$

d)  $y = \left| \frac{1}{4}x^2 - 9 \right|$

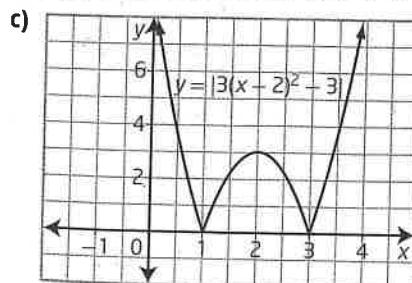
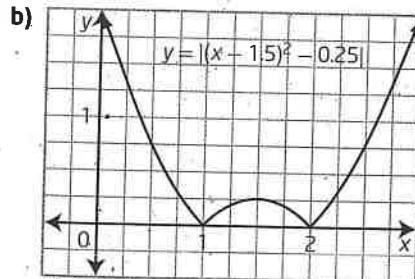
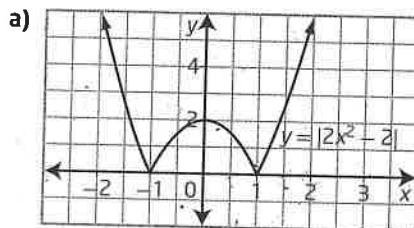
e)  $g(x) = |(x - 3)^2 + 1|$

f)  $h(x) = |-3(x + 2)^2 - 4|$

9. Write the piecewise function that represents each graph.



10. What piecewise function could you use to represent each graph of an absolute-value function?



11. Express each function as a piecewise function.

- a)  $y = |x - 4|$
- b)  $y = |3x + 5|$
- c)  $y = |-x^2 + 1|$
- d)  $y = |x^2 - x - 6|$

### Apply

12. Consider the function  $g(x) = |6 - 2x|$ .

- a) Create a table of values for the function using values of -1, 0, 2, 3, and 5 for  $x$ .
- b) Sketch the graph.
- c) Determine the domain and range for  $g(x)$ .
- d) Write the function in piecewise notation.

13. Consider the function  $g(x) = |x^2 - 2x - 8|$ .

- a) What are the  $y$ -intercept and  $x$ -intercepts of the graph of the function?
- b) Graph the function.
- c) What are the domain and range of  $g(x)$ ?
- d) Express the function as a piecewise function.

14. Consider the function  $g(x) = |3x^2 - 4x - 4|$ .

- a) What are the intercepts of the graph?
- b) Graph the function.
- c) What are the domain and range of  $g(x)$ ?
- d) What is the piecewise notation form of the function?

15. Raza and Michael are discussing the functions  $p(x) = 2x^2 - 9x + 10$  and  $q(x) = |2x^2 - 9x + 10|$ . Raza says that the two functions have identical graphs. Michael says that the absolute value changes the graph so that  $q(x)$  has a different range and a different graph from  $p(x)$ . Who is correct? Explain your answer.

4. a) 7 b) -5 c) 10 d) 13

5. Examples:

- a)  $|2.1 - (-6.7)| = 8.8$  b)  $|5.8 - (-3.4)| = 9.2$   
 c)  $|2.1 - (-3.4)| = 5.5$  d)  $|-6.7 - 5.8| = 12.5$

6. a) 10 b) -2.8 c) 5.25 d) 9 e) 17

7. Examples:

- a)  $|3 - 8| = 5$  b)  $|-8 - 12| = 20$   
 c)  $|9 - 2| = 7$  d)  $|15 - (-7)| = 22$   
 e)  $|a - b|$  f)  $|m - n|$

8.  $|7 - (-11)| + |9 - 7|; 34^\circ\text{C}$

9. Example:

$$|24 - 0| + |24 - 10| + |24 - 17| + |24 - 30| + |24 - 42| + |24 - 55| + |24 - 72|; 148 \text{ km}$$

10. 1743 miles

11. a) \$369.37

b) The net change is the change from the beginning point to the end point. The total change is all the changes in between added up.

12. a) 7.5 b) 90 c) 0.875

13. 4900 m or 4.9 km

14. a) 1649 ft b) 2325 ft

15. \$0.36

16. a) 6 km b) 9 km

17. a) The students get the same result of 90.66.

b) It does not matter the order in which you square something and take the absolute value of it.

c) Yes, because the result of squaring a number is the same whether it was positive or negative.

18. a) Michel looks at both cases; the argument is either positive or negative.

b) i)  $|x - 7| = \begin{cases} x - 7 & \text{if } x \geq 7 \\ 7 - x & \text{if } x < 7 \end{cases}$

ii)  $|2x - 1| = \begin{cases} 2x - 1 & \text{if } x \geq \frac{1}{2} \\ 1 - 2x & \text{if } x < \frac{1}{2} \end{cases}$

iii)  $|3 - x| = \begin{cases} 3 - x, & \text{if } x \leq 3 \\ x - 3, & \text{if } x > 3 \end{cases}$

iv)  $x^2 + 4$

19. Example: Changing +5 to -5 is incorrect.

Example: Change the sign so that it is positive.

20. 83 mm

21. Example: when you want just the speed of something and not the velocity

22. Example: signed because you want positive for up, negative for down, and zero for the top

23. a) 176 cm

- b) 4; 5; 2; 1; 4; 8; 1; 1; 2; 28 is the sum

- c) 3.11

- d) It means that most of the players are within 3.11 cm of the mean.

24. a) i)  $x = 1, x = -3$

ii)  $x = 1, x = -5$ ; you can verify by trying them in the equation.

- b) It has no zeros. This method can only be used for functions that have zeros.

25. Example: Squaring a number makes it positive, while the square root returns only the positive root.

## 7.2 Absolute Value Functions, pages 375 to 379

1. a)

$x$	$y =  f(x) $
-2	3
-1	1
0	1
1	3
2	5

b)

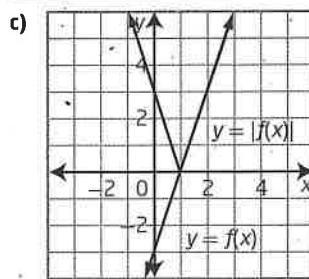
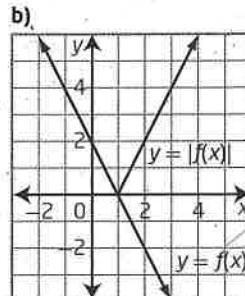
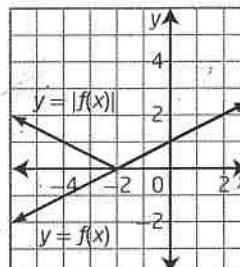
$x$	$y =  f(x) $
-2	0
-1	2
0	2
1	0
2	4

2.  $(-5, 8)$

3.  $x$ -intercept: 3;  $y$ -intercept: 4

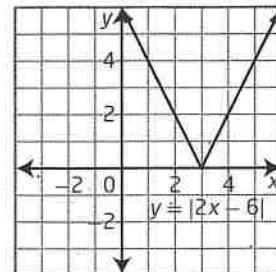
4.  $x$ -intercepts: -2, 7;  $y$ -intercept:  $\frac{3}{2}$

5. a)

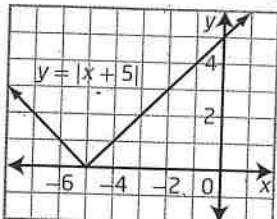


6. a)  $x$ -intercept: 3;  $y$ -intercept: 6;

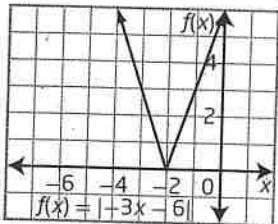
domain:  $\{x \mid x \in \mathbb{R}\}$ ; range:  $\{y \mid y \geq 0, y \in \mathbb{R}\}$



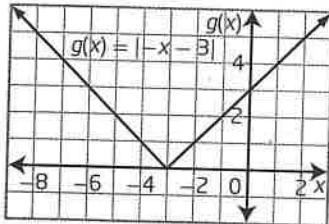
- b) x-intercept:  $-5$ ; y-intercept:  $5$ ;  
domain:  $\{x \mid x \in \mathbb{R}\}$ ; range:  $\{y \mid y \geq 0, y \in \mathbb{R}\}$



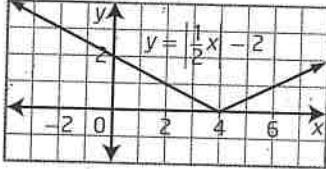
- c) x-intercept:  $-2$ ; y-intercept:  $6$ ;  
domain:  $\{x \mid x \in \mathbb{R}\}$ ; range:  $\{y \mid y \geq 0, y \in \mathbb{R}\}$



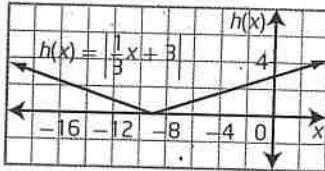
- d) x-intercept:  $-3$ ; y-intercept:  $3$ ;  
domain:  $\{x \mid x \in \mathbb{R}\}$ ; range:  $\{y \mid y \geq 0, y \in \mathbb{R}\}$



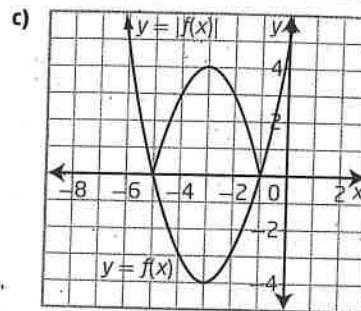
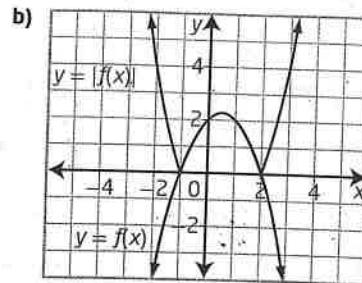
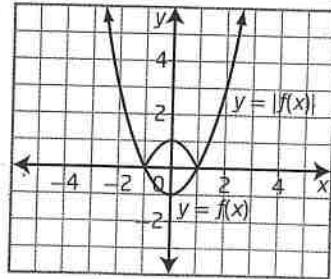
- e) x-intercept:  $4$ ; y-intercept:  $2$ ;  
domain:  $\{x \mid x \in \mathbb{R}\}$ ; range:  $\{y \mid y \geq 0, y \in \mathbb{R}\}$



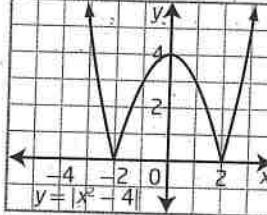
- f) x-intercept:  $-9$ ; y-intercept:  $3$ ;  
domain:  $\{x \mid x \in \mathbb{R}\}$ ; range:  $\{y \mid y \geq 0, y \in \mathbb{R}\}$



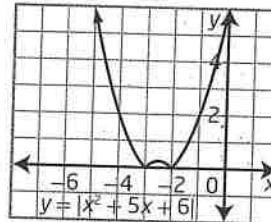
7. a)



8. a) x-intercepts:  $-2, 2$ ; y-intercept:  $4$ ;  
domain:  $\{x \mid x \in \mathbb{R}\}$ ; range:  $\{y \mid y \geq 0, y \in \mathbb{R}\}$



- b) x-intercepts:  $-3, -2$ ; y-intercept:  $6$ ;  
domain:  $\{x \mid x \in \mathbb{R}\}$ ; range:  $\{y \mid y \geq 0, y \in \mathbb{R}\}$



- c) x-intercepts:  $-2, 0.5$ ; y-intercept:  $2$ ;  
domain:  $\{x \mid x \in \mathbb{R}\}$ ; range:  $\{y \mid y \geq 0, y \in \mathbb{R}\}$

