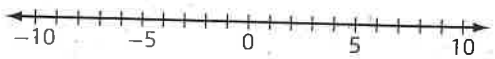
Check Your Understanding

Practise

1. Use the number line to geometrically solve each equation.

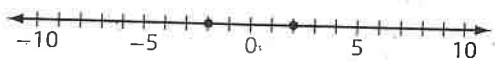


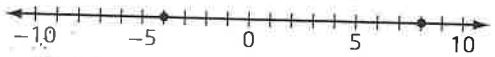
a) $|x| = 7$
b) $|x| + 8 = 12$
c) $|x| + 4 = 4$
d) $|x| = -6$

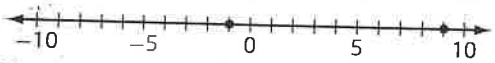
2. Solve each absolute value equation by graphing.

a) $|x - 4| = 10$
b) $|x + 3| = 2$
c) $6 = |x + 8|$
d) $|x + 9| = -3$

3. Determine an absolute value equation in the form $|ax + b| = c$ given its solutions on the number line.

a)


b)


c)


4. Solve each absolute value equation algebraically. Verify your solutions.

a) $|x + 7| = 12$
b) $|3x - 4| + 5 = 7$
c) $2|x + 6| + 12 = -4$
d) $-6|2x - 14| = -42$

5. Solve each equation.

a) $|2a + 7| = a - 4$
b) $|7 + 3x| = 11 - x$
c) $|1 - 2m| = m + 2$
d) $|3x + 3| = 2x - 5$
e) $3|2a + 7| = 3a + 12$

6. Solve each equation and verify your solutions graphically.

a) $|x| = x^2 + x - 3$
b) $|x^2 - 2x + 2| = 3x - 4$
c) $|x^2 - 9| = x^2 - 9$
d) $|x^2 - 1| = x$
e) $|x^2 - 2x - 16| = 8$

Apply

7. Bolts are manufactured at a certain factory to have a diameter of 18 mm and are rejected if they differ from this by more than 0.5 mm.

a) Write an absolute value equation in the form $|d - a| = b$ to describe the acceptance limits for the diameter, d, in millimetres, of these bolts, where a and b are real numbers.

b) Solve the resulting absolute value equation to find the maximum and minimum diameters of the bolts.

8. One experiment measured the speed of light as 299 792 456.2 m/s with a measurement uncertainty of 1.1 m/s.

a) Write an absolute value equation in the form $|c - a| = b$ to describe the measured speed of light, c, metres per second, where a and b are real numbers.

b) Solve the absolute value equation to find the maximum and minimum values for the speed of light for this experiment.

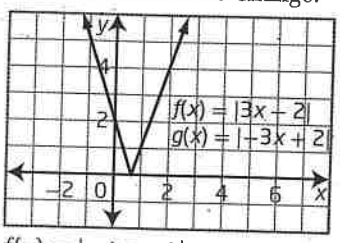
9. In communities in Nunavut, aviation fuel is stored in huge tanks at the airport. Fuel is re-supplied by ship yearly. The fuel tank in Kugaaruk holds 50 000 L. The fuel re-supply brings a volume, V, in litres, of fuel plus or minus 2000 L.

a) Write an absolute value equation in the form $|V - a| = b$ to describe the limits for the volume of fuel delivered, where a and b are real numbers.

b) Solve your absolute value equation to find the maximum and minimum volumes of fuel.

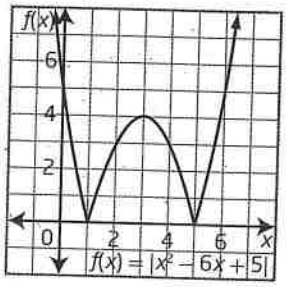
17. The distance travelled is 13 m.

18. a) The two graphs are identical. They are identical because one is the negative of the other but since they are in absolute value brackets there is no change.



b) $f(x) = |-4x - 3|$

19. $f(x) = |-x^2 + 6x - 5|$



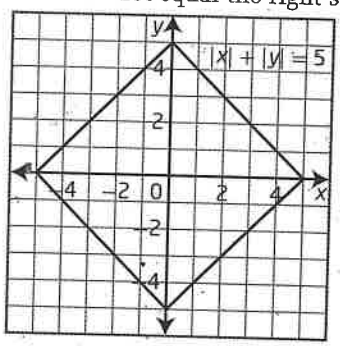
20. $a = -4$, $b = 6$ or $a = 4$, $b = -6$

21. $b = 4$; $c = -12$

22. Example: The square of something is always positive, so taking the absolute value does nothing.

23. Example: No, it is not true for all $x, y \in R$. For instance, if x and y are of different sign the left side will not equal the right side.

24.



25.

Case	$\lvert x\rvert\lvert y\rvert$	$\lvert xy\rvert$
$x \geq 0, y \geq 0$	xy	xy
$x \geq 0, y < 0$	$x(-y)$	$-xy$
$x < 0, y \geq 0$	$(-x)y$	$-xy$
$x < 0, y < 0$	$(-x)(-y)$	xy

26. Example: They have the same shape but different positions.

27. Example: Graph the functions, taking care to allow them only in their specified domain.

28. If the discriminant is less than or equal to 0 and $a > 0$, then the graphs will be equivalent.

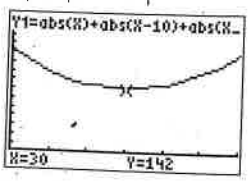
29. Examples:

Step 1 Yes.

Step 2 Absolute value is needed because the facility could be to the east or west of each town.

$$\text{total} = |x| + |x - 10| + |x - 17| + |x - 30| + |x - 42| + |x - 55| + |x - 72|$$

Step 3 x: [0, 60, 10]
y: [−30, 300, 20]



Step 4 The point (30, 142) on the graph shows that there is a point that minimizes the distance to each city. The point represents a place 30 km east of Allenby and results in a total distance from all towns of 142 km.

30. a) $y = |(x - 3)^2 + 7|$ b) $y = \left|\frac{4}{5}(x + 3)^2\right|$
c) $y = |-x^2 - 6|$ d) $y = |5(x + 3)^2 + 3|$

7.3 Absolute Value Equations, pages 389 to 391

1. a) $x = -7$, $x = 7$ b) $x = -4$, $x = 4$
c) $x = 0$ d) no solution
2. a) $x = -6$, $x = 14$ b) $x = -5$, $x = -1$
c) $x = -14$, $x = -2$ d) no solution
3. a) $|x| = 2$ b) $|x - 2| = 6$
c) $|x - 4| = 5$
4. a) $x = -19$, $x = 5$ b) $x = \frac{2}{3}$, $x = 2$
c) no solution d) $x = 3.5$, $x = 10.5$
5. a) no solution b) $x = -9$, $x = 1$
c) $m = -\frac{1}{3}$, $m = 3$ d) no solution
e) $a = -\frac{11}{3}$, $a = -3$
6. a) $x = -3$, $x = \sqrt{3}$ b) $x = 2$, $x = 3$
c) $x \leq -3$ or $x \geq 3$
d) $x = \frac{1 + \sqrt{5}}{2}$, $x = \frac{\sqrt{5} - 1}{2}$
e) $x = -4$, $x = -2$, $x = 4$, $x = 6$
7. a) $|d - 18| = 0.5$
b) 17.5 mm and 18.5 mm are allowed
8. a) $|c - 299\ 792\ 456.2| = 1.1$
b) 299 792 455.1 m/s or 299 792 457.3 m/s
9. a) $|V - 50\ 000| = 2000$ b) 48 000 L, 52 000 L
10. a) 2.2, 11.8 b) $|x - 7| = 4.8$
11. a) 66.5 g b) 251 mL and 265 mL
12. a) perigee: 356 400 km; apogee: 406 700 km
b) Example: The moon is usually around 381 550 km away plus or minus 25 150 km.
13. a) greater than or equal to zero
b) less than or equal to zero
14. a) $x = \frac{b + c}{a}$ if $x \geq 0$, $x = \frac{-b - c}{a}$ if $x < 0$; $b + c \geq 0$, $a \neq 0$
b) $x = b + c$ if $x \geq b$, $x = b - c$ if $x < b$; $c \geq 0$