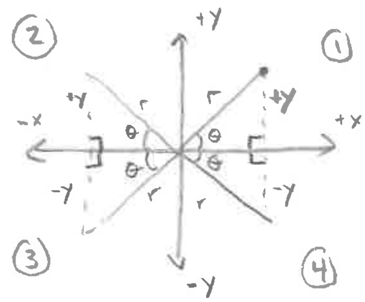
ASSIGNMENT # 3

Ch. 2.2 Trig Ratios of Sin, Cos, Tan

① $\sin\theta$ = Negative Ratio



$\sin\theta = \frac{y}{r}$

r = always positive

$\sin\theta$ = Negative Ratio when opposite is negative (-y)

since r = always positive

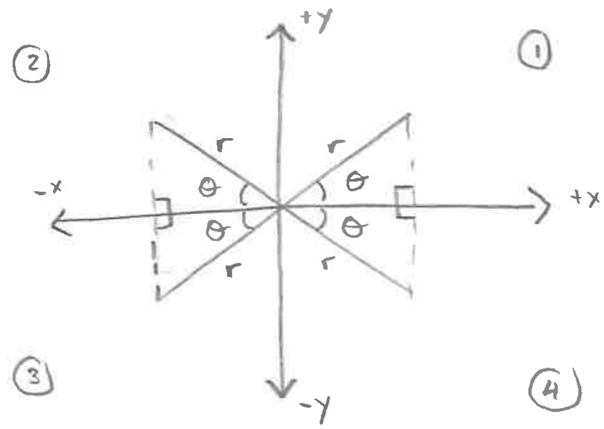
$\boxed{\text{Quadrant } 3, 4}$

$\sin\theta$ = Positive Ratio when opposite is positive (+y)

since r = always positive

$\boxed{\text{Quadrant } 1, 2}$

(2) $\cos\theta$ = Negative Ratio



$\cos\theta = \frac{x}{r}$

r = always positive

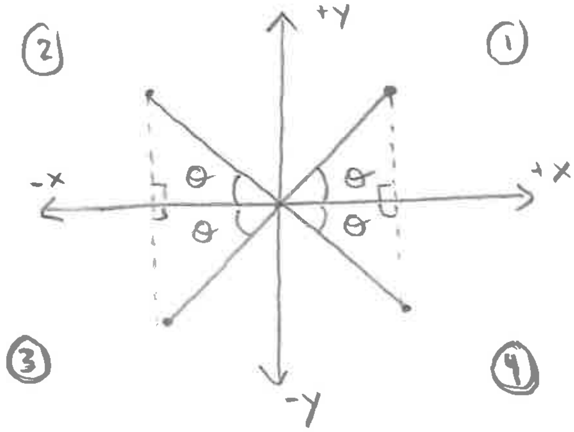
$\cos\theta$ = Negative Ratio when ADJACENT is negative (-x) since r = always positive

$\boxed{\text{QUADRANT } 2, 3}$

$\cos\theta$ = Positive Ratio when ADJACENT is positive (+x) since r = always positive

$\boxed{\text{QUADRANT } 1, 4}$

(3) $\tan\theta$ = Negative Ratio



In Quadrant 1 : $\tan\theta = \frac{+y}{+x}$ = Positive

In Quadrant 2 : $\tan\theta = \frac{+y}{-x}$ = Negative

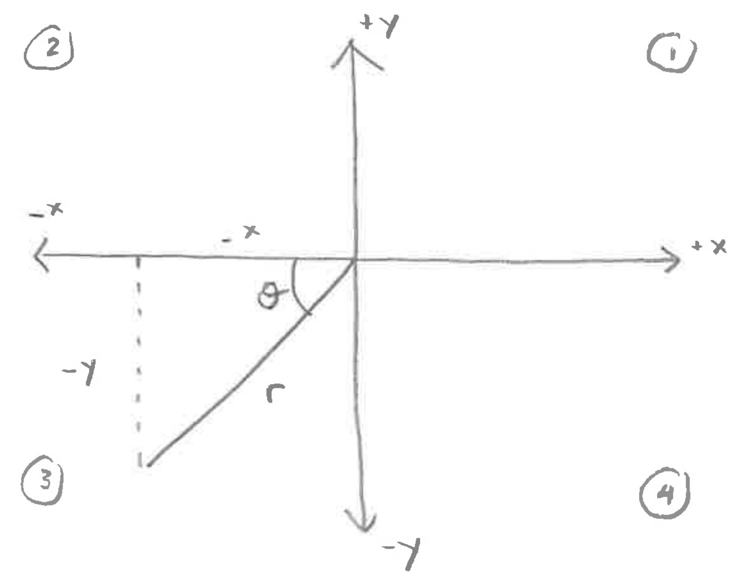
In Quadrant 3 : $\tan\theta = \frac{-y}{-x}$ = Positive

In Quadrant 4 : $\tan\theta = \frac{-y}{+x}$ = Negative

NEGATIVE : $\boxed{\text{QUAD } 2, 4}$

POSITIVE : $\boxed{\text{QUAD } 1, 3}$

(4) If θ is in Quadrant 3

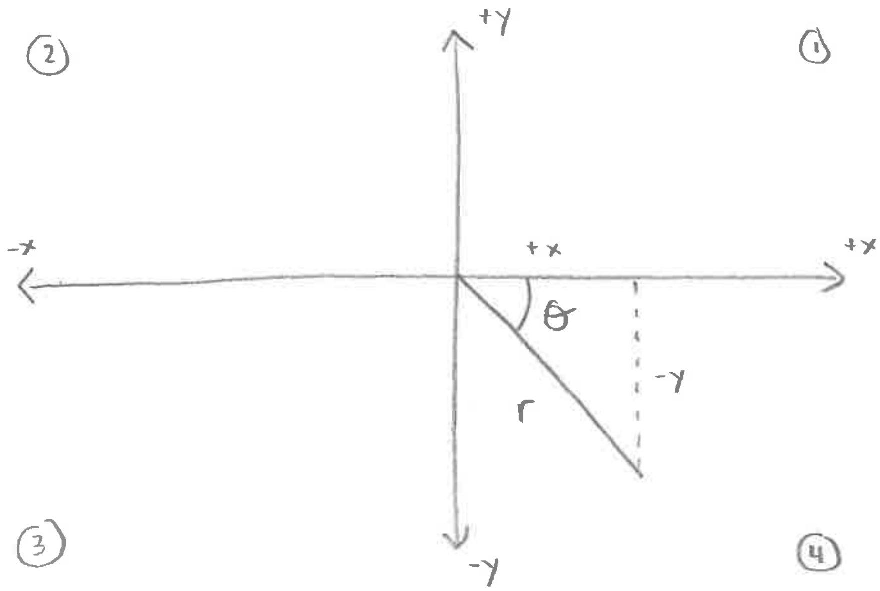


$\sin\theta = \frac{-y}{r}$ NEGATIVE

$\cos\theta = \frac{-x}{r}$ NEGATIVE

$\tan\theta = \frac{-y}{-x}$ POSITIVE

⑤ If θ is in QUADRANT 4

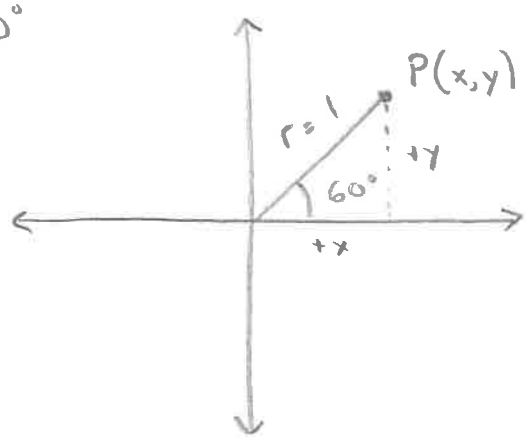


$$\sin\theta = \frac{-y}{r} \quad \text{NEGATIVE}$$

$$\cos\theta = \frac{+x}{r} \quad \text{POSITIVE}$$

$$\tan\theta = \frac{-y}{+x} \quad \text{NEGATIVE}$$

7a $60°$



$\theta_R = 60°$

$$\boxed{P(x,y) = P(\overset{x}{\cos\theta_R}, \overset{y}{\sin\theta_R})}$$

We can use this rule since we are on the Unit Circle w/ r=1

$$\sin\theta = \frac{+y}{r}$$

$$\sin\theta = \frac{y}{1} \rightarrow y = 1 \times \sin\theta$$

$$y = \sin\theta$$

$$\boxed{P = (\cos 60, \sin 60)}$$

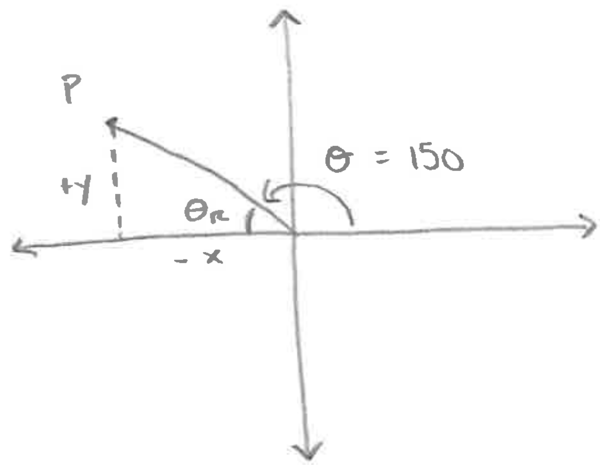
$$\cos\theta = \frac{+x}{r}$$

$$\cos\theta = \frac{+x}{1} \rightarrow x = 1\cos\theta$$

$$x = \cos\theta$$

Both x and y are positive in QUAD I

(7b) 150°



$\theta_R = 180° - 150°$

$\theta_R = 30°$

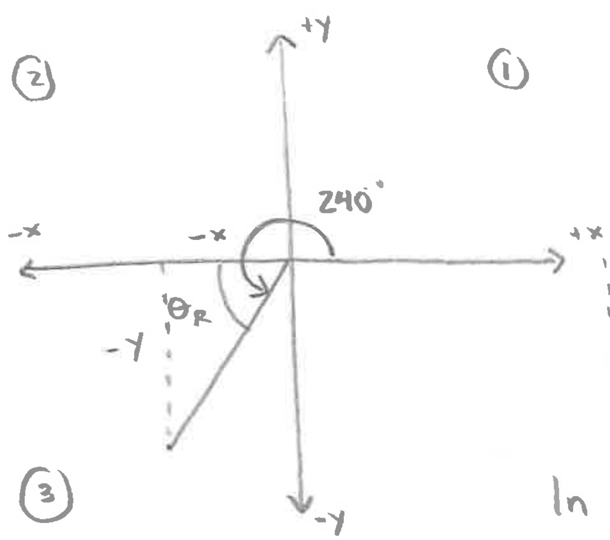
$P(x,y) = P(\cos\theta_R, \sin\theta_R)$

In quadrant 2, y is positive and x is negative

So $P = (-\cos\theta_R, \sin\theta_R)$

$$\boxed{P = (-\cos 30, \sin 30)}$$

(7c) 240°



$\theta_R = 240° - 180°$

$\theta_R = 60°$

$P(x,y) = P(\cos\theta_R, \sin\theta_R)$

In Quadrant 3, y is negative and x is negative

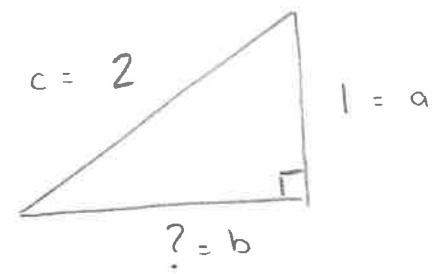
So, $P = (-\cos\theta_R, -\sin\theta_R)$

$$\boxed{P = (-\cos 60, -\sin 60)}$$

(8a) $\sin\theta = 0.5$ $\qquad$ $\sin\theta = \frac{1}{2}$ (this lets us know the opp and hyp)

$\cos\theta = ?$

$\tan\theta = ?$



Using Pythagorean Theory

$$a^2 + b^2 = c^2$$

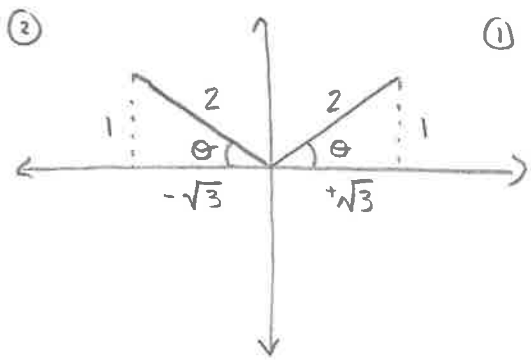
$$1^2 + b^2 = 2^2$$

$$1 + b^2 = 4$$

$$b^2 = 4 - 1$$

$$b^2 = 3 \quad \text{(take square root of both sides)}$$

$$b = \sqrt{3}$$



$\sin\theta = +\frac{1}{2}$ $(+y)$

Positive Ratio in Q1 and Q_2

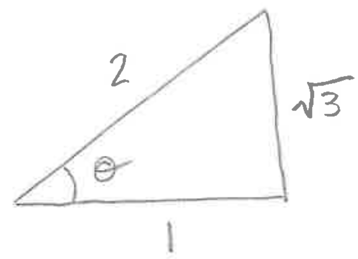
$$\boxed{\cos\theta = -\frac{\sqrt{3}}{2}, +\frac{\sqrt{3}}{2}}$$

$$\boxed{\tan\theta = \frac{-1}{\sqrt{3}}, +\frac{1}{\sqrt{3}}}$$

8c) $\tan\theta = -\sqrt{3}$ must be the same as $\tan\theta = \frac{-\sqrt{3}}{1}$

$\cos\theta =$

$\sin\theta =$



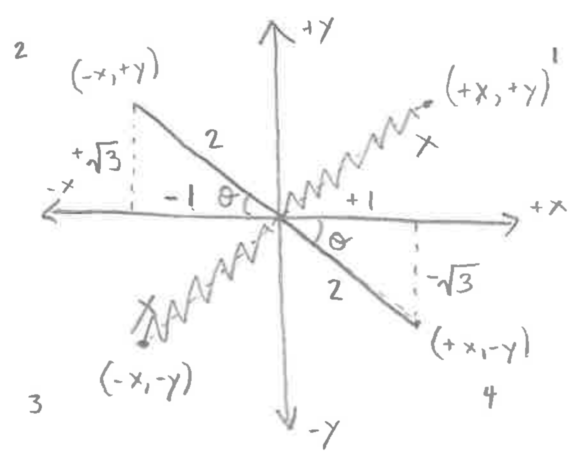
$a^2 + b^2 = c^2$

$\sqrt{3}^2 + 1^2 = 2^2$

$c^2 = 4$

$c = 2$

Since $\frac{-\sqrt{3}}{1}$ is a negative ratio, either the 1 **OR** the $\sqrt{3}$ must be negative. If both are negative, the ratio is positive.



Therefore,

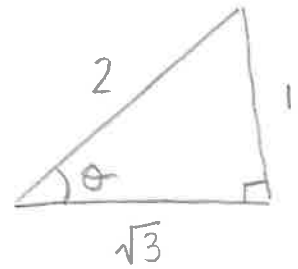
$$\boxed{\cos\theta = \frac{-1}{2}, \frac{1}{2}}$$

$$\boxed{\sin\theta = \frac{+\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}}$$

(8f) $\tan\theta = \frac{1}{\sqrt{3}}$

$\cos\theta = ?$

$\sin\theta = ?$



$a^2 + b^2 = c^2$

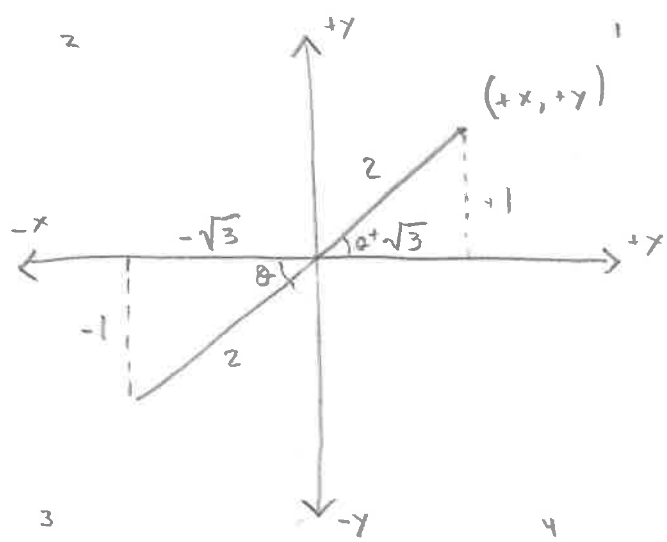
$1^2 + \sqrt{3}^2 = c^2$

$1 + 3 = c^2$

$c^2 = 4$

$c = 2$

Since $\frac{1}{\sqrt{3}}$ is a positive ratio, x and y must both be positive, or both be negative



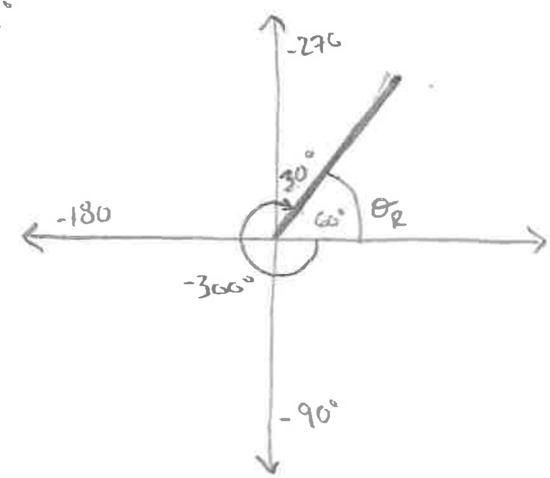
Therefore,

$$\boxed{\cos\theta = \frac{-\sqrt{3}}{2}, \frac{\sqrt{3}}{2}}$$

$$\boxed{\sin\theta = \frac{-1}{2}, \frac{1}{2}}$$

Ch. 2.1 Angles in Standard Position

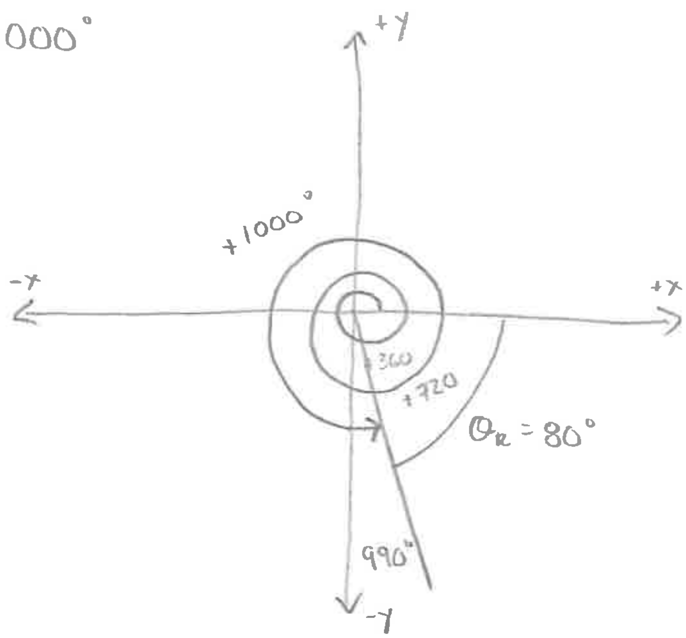
(1b) $-300°$



Since it's negative we move clockwise

$\theta_R = 60°$

(1f) $1000°$



Since it's positive move counterclockwise

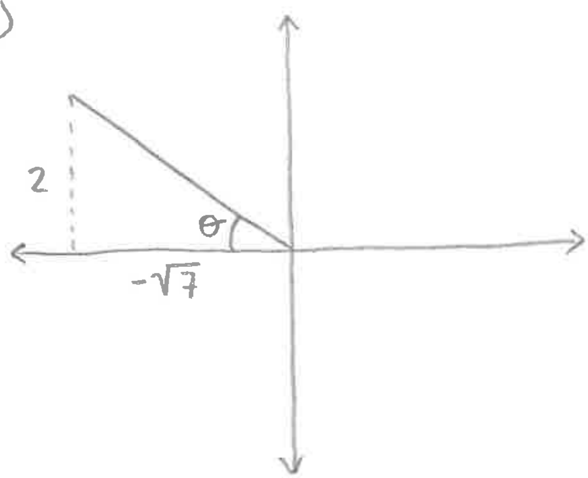
$360 + 360 = 720$

$1000 - 720 = 280$

$\theta_R = 80°$

(12) If Tan $= -\frac{2}{\sqrt{7}}$, Angle θ is in standard position, and terminal arm in quadrant II

(11)



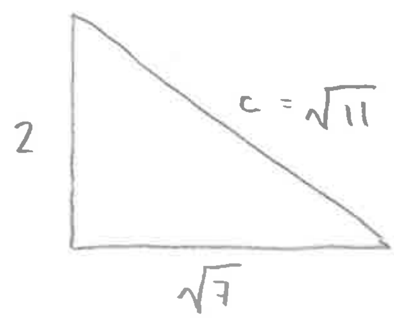
For $\tan\theta = \frac{-2}{\sqrt{7}}$

either $y = 2$, or $x = \sqrt{7}$ must be negative.

If Q2, only x can be Negative

$y = +2$

$x = -\sqrt{7}$



$\sqrt{7}^2 + 2^2 = c^2$

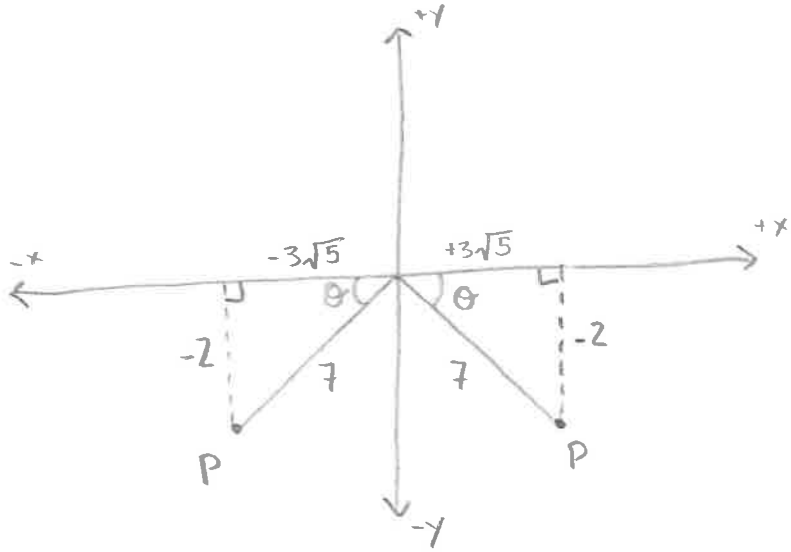
$7 + 4 = c^2$

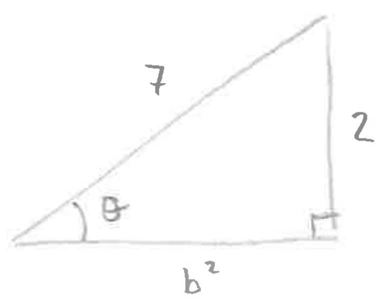
$11 = c^2$

$c = \sqrt{11}$

$$\boxed{\cos\theta = -\frac{\sqrt{7}}{\sqrt{11}}}$$

(13) If $\sin\theta = \frac{-2}{7}$, must be in either Quadrant 3 or Quadrant 4, since $y = -2$





$b^2 = c^2 - a^2$

$b^2 = 7^2 - 2^2$

$b^2 = 49 - 4$

$b^2 = 45$

$b = 3\sqrt{5}$ $\qquad b = \sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$

Possible Coordinates for P

$P(-3\sqrt{5}, -2)$ or $P(3\sqrt{5}, -2)$

$\boxed{\cos\theta = \frac{-3\sqrt{5}}{7}, \frac{+3\sqrt{5}}{7}}$

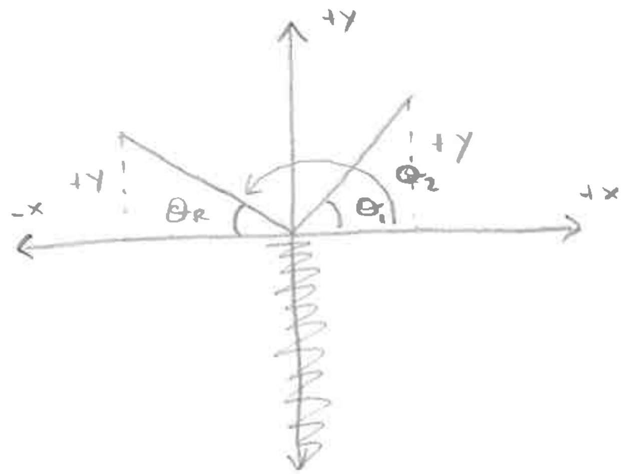
$\boxed{\tan\theta = \frac{+2}{+3\sqrt{5}}, -\frac{2}{3\sqrt{5}}}$

Ch. 2.3 SINE LAW

(1a) $\sin\theta = 0.25$

$\sin\theta$ = positive

$0 \leq \theta \leq 180$ means that we are only concerned with Q1 and Q2



$\theta = \sin^{-1}(0.25)$

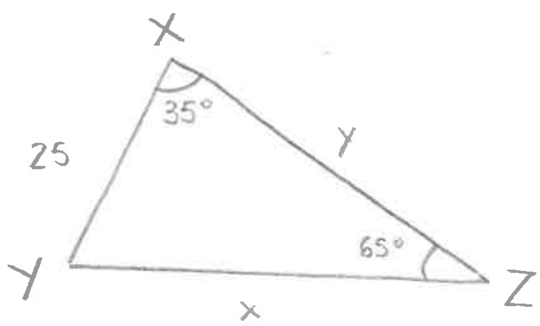
$\theta = 14.5^\circ$

OR

$\theta = 180^\circ - 14.5^\circ$
$= 165.5^\circ$

$$\boxed{\theta = 14.5^\circ \text{ or } 165.5^\circ}$$

(2a)



SINE LAW $= \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

In our case I labelled opposite angles and sides the same letter. LowerCase for side, UPPERCASE for angle

$$\frac{z}{\sin Z} = \frac{y}{\sin Y} = \frac{x}{\sin X}.$$

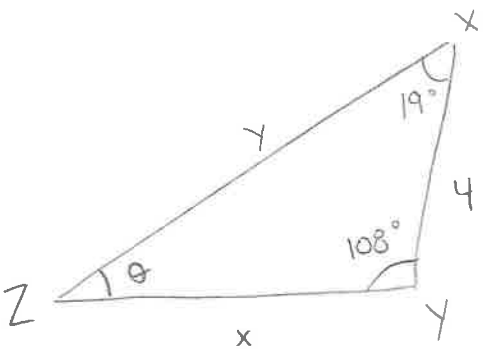
$$\frac{25}{\sin 65} = \cancel{\frac{y}{\sin Y}} = \frac{x}{\sin 35}$$

$$\frac{25}{\sin 65} = \frac{x}{\sin 35}$$ One equation, One Unknown!

$$x = \frac{25 \sin 35}{\sin 65}$$

$$\boxed{x = 15.8}$$

(2c)



$\theta = 180° - 19° - 108°$

$\theta = 53°$

$$\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z}$$

$$\frac{x}{\sin 19} = \cancel{\frac{y}{\sin 108}} = \frac{4}{\sin 53}$$

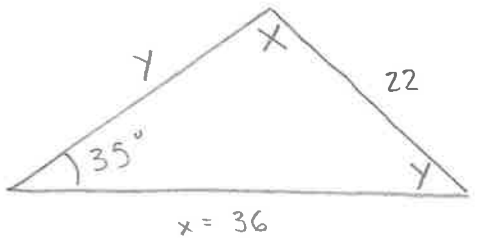
$$\frac{x}{\sin 19} = \frac{4}{\sin 53}$$

This pair lets us solve for x.

$$x = \frac{4 \sin 19}{\sin 53}$$

$$\boxed{x = 1.6}$$

(2f)



$$\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z}$$

$$\frac{36}{\sin X} = \frac{y}{\sin Y} = \frac{22}{\sin 35}$$

Using these two will give me 1 unknown.

$$\cancel{\sin X} \times \frac{36}{\cancel{\sin X}} = \frac{22}{\sin 35} \times \sin X$$

solve for sinX

$$\frac{\sin 35}{22} \times 36 = \frac{\cancel{22} \sin X}{\cancel{\sin 35}} \times \frac{\cancel{\sin 35}}{\cancel{22}}$$

$$\sin X = \frac{36 \sin 35}{22}$$

$$X = \sin^{-1}\left(\frac{36 \sin 35}{22}\right)$$

$$X = \sin^{-1}(0.939)$$

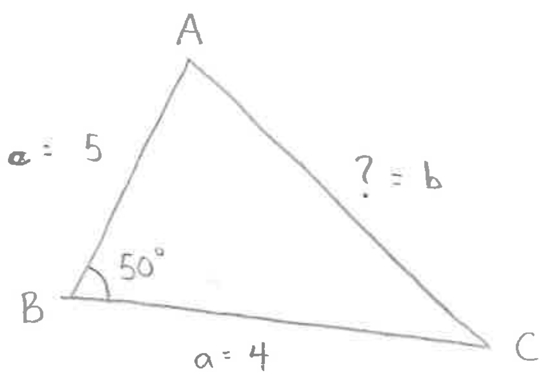
$$\boxed{X = 69.8^\circ}$$

$$Y = 180^\circ - 35^\circ - 69.8^\circ$$

$$\boxed{Y = 75.2^\circ}$$

Ch. 2.4 Cosine Law

1a



Cosine Law

$$b^2 = a^2 + c^2 - 2ac\cos B$$

Only unknown is b^2

Solve

$$b^2 = (4)^2 + (5)^2 - 2(4)(5)\cos 50$$

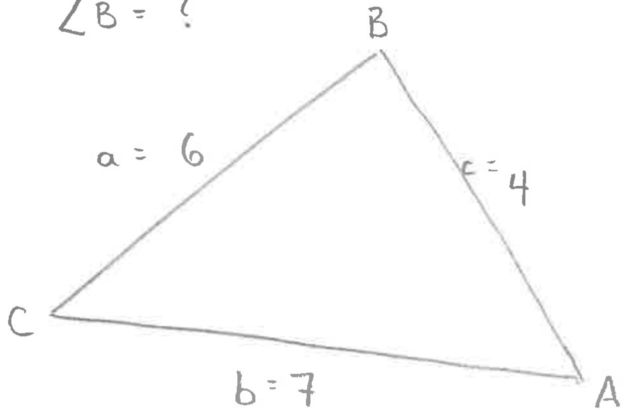
$$b^2 = 41 - 40\cos 50$$

$$b^2 = 15.3$$

$$b = \sqrt{15.3}$$

$$\boxed{b = 3.9}$$

(1c) $\angle B = ?$



Cosine Law

$$b^2 = a^2 + c^2 - 2ac \cos B$$

Solve for $\cos B$

$$\frac{b^2 - a^2 - c^2}{-2ac} = \frac{-\cancel{2ac} \cos B}{\cancel{-2ac}}$$

$$\cos B = \frac{b^2 - a^2 - c^2}{-2ac}$$

$$\cos B = \frac{7^2 - 6^2 - 4^2}{-2(6)(4)}$$

$$\cos B = \frac{-3}{-48}$$

$$\cos B = 0.0625$$

$$B = \cos^{-1}(0.0625)$$

$$\boxed{B = 86.4^\circ}$$