

$$\textcircled{1} \quad a - b = 10$$

$$\boxed{a = 10 + b}$$

$$P = a \times b$$

$$P = (10 + b) \times b$$

$$\boxed{P = 10b + b^2}$$

$$a \times b = P(\text{min})$$

Complete the Square

$$P = b^2 + 10b + 0$$

$$P = b^2 + 10b + 25 - 25$$

$$\boxed{P = (b + 5)^2 - 25}$$

$$\boxed{P(\text{min}) = -25}$$

$$\boxed{b = -5}$$

$$\boxed{a = 5}$$

$$a = 10 + b$$

$$a = 10 - 5$$

$$\textcircled{2} \quad a + b = 12$$

$$a \times b = P(\text{max})$$

$$P = (12 - b)b$$

$$P = 12b - b^2$$

$$P = -b^2 + 12b$$

$$P = -[b^2 - 12b + 36 - 36]$$

$$P = -[(b - 6)^2 - 36]$$

$$\boxed{P = -(b - 6)^2 + 36}$$

Complete the square

$$\boxed{P(\text{max}) = 36}$$

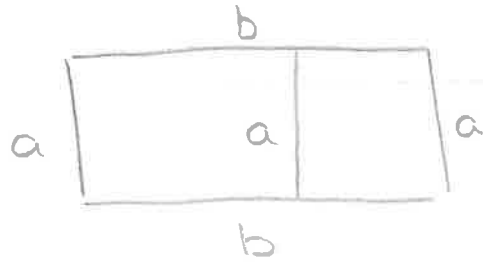
$$\boxed{b = 6}$$

$$a = 12 - b$$

$$a = 12 - 6$$

$$\boxed{a = 6}$$

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$$3a + 2b = 800$$

$$2b = 800 - 3a$$

$$A = a \times b$$

$$b = 400 - \frac{3}{2}a$$

$$b = 400 - 1.5a$$

$$A = a \times b$$

$$A = a \times (400 - 1.5a)$$

$$A = 400a - 1.5a^2$$

$$A = -1.5a^2 + 400a$$

Complete Square

$$A = -1.5 \left[a^2 - 266.67a + 17777.78 - 17777.78 \right]$$

$$A = -1.5 \left[(a - 133.33)^2 - 17777.78 \right]$$

$$A = -1.5(a - 133.33)^2 + 26666.7$$

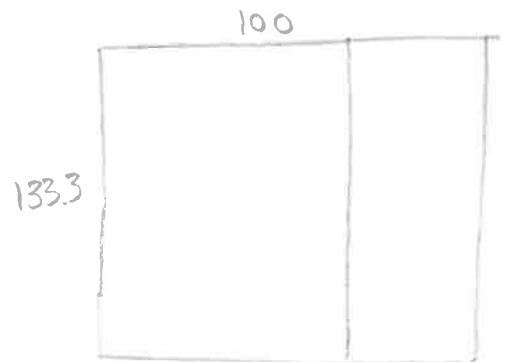
$$A_{\max} = 26666.7 \text{ m}^2$$

$$a = 133.33 \text{ m}$$

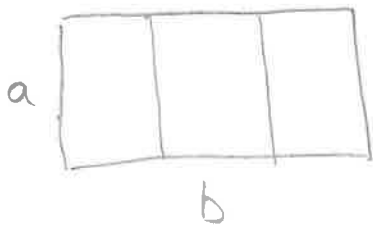
$$b = 400 - 1.5a$$

$$b = 400 - 1.5(133.33)$$

$$b = 100 \text{ m}$$



④



$$4a + 2b = 800$$

$$A = a \times b$$

$$2b = 800 - 4a$$

$$b = \frac{800}{2} - \frac{4a}{2}$$

$$b = 400 - 2a$$

$$A = a \times b$$

$$A = a \times (400 - 2a)$$

$$A = 400a - 2a^2$$

Complete the Square

$$A = -2a^2 + 400a$$

$$A = -2(a^2 - 200a + 10000 - 10000)$$

$$A = -2((a - 100)^2 - 10000)$$

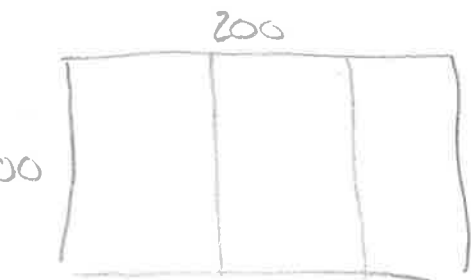
$$A = -2(a - 100)^2 + 20000$$

$$A_{\max} = 20000 \text{ m}^2$$

$$a = 100 \text{ m}$$

$$b = 400 - 2a$$

$$b = 400 - 2(100)$$



$$b = 200 \text{ m}$$

⑤ $R = \# \text{ Tickets} \times \text{Price per ticket}$

$$R = T \times P$$

$T = \# \text{ tickets sold}$

$P = \text{price per ticket}$

$R = \text{revenue}$

$$T = 200 - 5 \left(\frac{P - 50}{10} \right)$$

for every \$10 increase in price 5 fewer tickets sold from 200.

P is the ticket price

$P - 50$ is the total price increase from 50.

$\frac{P - 50}{10}$ are the number of \$10 increases.

$5 \times \left(\frac{P - 50}{10} \right)$ are the number of fewer tickets sold.

$$R = T \times P$$

$$R = \left[200 - 5 \left(\frac{P - 50}{10} \right) \right] \times P$$

$$R = 200P - \frac{5P^2 + 250P}{10}$$

$$R = 200P - 0.5P^2 + 25P$$

$$R = -0.5P^2 + 225P$$

Complete the Square

$$R = -0.5 \left[P^2 - 112.5P + 3164 - 3164 \right]$$

$$R = -0.5 \left[(P - 56.25)^2 - 3164 \right]$$

$$R = -0.5(P - 56.25)^2 + 1582$$

$$\text{Max Revenue} = \$1582$$

$$\text{Price} = \$56.25$$

$$T = 196.9 \approx 197$$

⑥ $R = T \times P$ $R = \text{Revenue}$
 $T = \# \text{ Tickets sold}$
 $P = \text{Price per ticket}$

$$T = 400 - 20 \left[\frac{P - 30}{3} \right]$$

for every 3 dollar increase from 30 dollars of P you lose 20 tickets.

$$R = P \times T$$

$$R = P \times \left[400 - 20 \left(\frac{P - 30}{3} \right) \right]$$

$$R = 400P - \frac{20}{3}P^2 + \frac{600}{3}P$$

$$R = 400P - 6.67P^2 + 200P$$

$$R = -6.67P^2 + 600P$$

Complete Square

$$R = -6.67 [P^2 - 90P]$$

$$R = -6.67 [P^2 - 90P + 2025 - 2025]$$

$$R = -6.67 [(P - 45)^2 - 2025]$$

$$R = -6.67 (P - 45)^2 + 13506.8$$

$$R_{\max} = \$13507$$

$$P = \$45$$

$$T = 400 - 20 \left[\frac{45 - 30}{3} \right]$$

$$T = 300 \text{ tickets}$$

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$$R = B \times P$$

R = Revenue

B = Bikes Sold #

P = Bike Price

$$B = 70 - 10 \left(\frac{P - 300}{25} \right)$$

$$R = P \times B$$

$$R = P \times \left[70 - 10 \left(\frac{P - 300}{25} \right) \right]$$

$$R = 70P - \frac{10P^2}{25} + \frac{3000P}{25}$$

$$R = 70P - 0.4P^2 + 120P$$

$$R = -0.4P^2 + 190P$$

Complete Square

$$R = -0.4 \left[P^2 - 475P \right]$$

$$R = -0.4 \left[P^2 - 475P + 56406 - 56406 \right]$$

$$R = -0.4 \left[(P - 237.5)^2 - 56406 \right]$$

$$R = -0.4 (P - 237.5)^2 + 22563$$

$$R_{max} = \$22563$$

$$P = \$237.5$$

$$B = 70 - 10 \left(\frac{237.5 - 300}{25} \right)$$

$$B = 95$$

⑧



$$P = 60\text{m} \quad P = 2l + 2w$$

$$A = l \times w$$

$$2l + 2w = 60$$

$$2w = 60 - 2l$$

$$w = 30 - l$$

$$A = l \times w$$

$$A = (30 - l) \times l$$

$$A = 30l - l^2$$

$$A = -l^2 + 30l$$

Complete Square

$$A = - [l^2 - 30l + 225 - 225]$$

$$A = - [(l - 15)^2 - 225]$$

$$A = - (l - 15)^2 + 225$$

$$A_{\max} = 225\text{m}^2$$

$$l = 15\text{m}$$

$$w = 15\text{m}$$