Example 4 Solving Problems Using the Exponent Laws

A sphere has volume 425 m³.
What is the radius of the sphere to the nearest tenth of a metre?

SOLUTION

The volume V of a sphere with radius r is given by the formula: $V = \frac{4}{3}\pi r^3$. Substitute $V = 425$, then solve for r.

$425 = \frac{4}{3}\pi r^3$ Multiply each side by 3.

$3(425) = 3\left(\frac{4}{3}\pi r^3\right)$

$1275 = 4\pi r^3$ Divide each side by 4π.

$\frac{1275}{4\pi} = \frac{4\pi r^3}{4\pi}$

$\frac{1275}{4\pi} = r^3$ To solve for r, take the cube root of each side by raising each side to the one-third power.

$\left(\frac{1275}{4\pi}\right)^{\frac{1}{3}} = (r^3)^{\frac{1}{3}}$ Use the power of a power law.

$\left(\frac{1275}{4\pi}\right)^{\frac{1}{3}} = r$

$r = 4.6640\ldots$

The radius of the sphere is approximately 4.7 m.

CHECK YOUR UNDERSTANDING

4. A cone with height and radius equal has volume 18 cm³. What are the radius and height of the cone to the nearest tenth of a centimetre?

[Answer: approximately 2.6 cm]

How do you know that the length of the radius is an irrational number?

Discuss the Ideas

1. Suppose you want to evaluate an algebraic expression for particular values of the variables. Why might it be helpful to simplify the expression first?
2. When you simplify an expression, how do you know which exponent law to apply first?

Exercises

A

3. Simplify.

a) $x^3 \cdot x^4$ b) $a^2 \cdot a^{-5}$
c) $b^{-3} \cdot b^5$ d) $m^2 \cdot m^{-3}$

4. Write as a single power.

a) $0.5^2 \cdot 0.5^3$ b) $0.5^2 \cdot 0.5^{-3}$
c) $\frac{0.5^2}{0.5^3}$ d) $\frac{0.5^2}{0.5^{-3}}$

5. Simplify.

a) $\frac{x^4}{x^2}$ b) $\frac{x^2}{x^5}$
c) $n^6 \div n^5$ d) $\frac{a^2}{a^6}$

6. Simplify.

a) $(n^2)^3$ b) $(z^2)^{-3}$
c) $(n^{-4})^{-3}$ d) $(c^{-2})^2$

7. Write as a single power.

a) $\left[\left(\frac{3}{5}\right)^3\right]^4$ b) $\left[\left(\frac{3}{5}\right)^3\right]^{-4}$

c) $\left[\left(\frac{3}{5}\right)^{-3}\right]^{-4}$ d) $\left[\left(-\frac{3}{5}\right)^{-3}\right]^{-4}$

8. Simplify.

a) $\left(\frac{a}{b}\right)^2$ b) $\left(\frac{n^2}{m}\right)^3$

c) $\left(\frac{c^2}{d^2}\right)^{-4}$ d) $\left(\frac{2b}{5c}\right)^2$

e) $(ab)^2$ f) $(n^2m)^3$

g) $(c^3d^2)^{-4}$ h) $(xy^{-1})^3$

B

9. Simplify. State the exponent law you used.

a) $x^{-3} \cdot x^4$ b) $a^{-4} \cdot a^{-1}$

c) $b^4 \cdot b^{-3} \cdot b^2$ d) $m^8 \cdot m^{-2} \cdot m^{-6}$

e) $\frac{x^{-5}}{x^2}$ f) $\frac{s^5}{s^{-5}}$

g) $\frac{b^{-8}}{b^{-3}}$ h) $\frac{t^{-4}}{t^{-4}}$

10. Evaluate.

a) $1.5^{\frac{3}{2}} \cdot 1.5^{\frac{1}{2}}$ b) $\left(\frac{3}{4}\right)^{\frac{3}{4}} \cdot \left(\frac{3}{4}\right)^{\frac{5}{4}}$

c) $(-0.6)^{\frac{1}{3}} \cdot (-0.6)^{\frac{5}{3}}$ d) $\left(\frac{4}{5}\right)^{\frac{4}{3}} \cdot \left(\frac{4}{5}\right)^{-\frac{4}{3}}$

e) $\frac{0.6^{\frac{1}{2}}}{0.6^{\frac{3}{2}}}$ f) $\frac{\left(-\frac{3}{8}\right)^{\frac{2}{3}}}{\left(-\frac{3}{8}\right)^{-\frac{1}{3}}}$

g) $\frac{0.49^{\frac{5}{2}}}{0.49^4}$ h) $\frac{0.027^{\frac{5}{3}}}{0.027^{\frac{4}{3}}}$

11. Simplify. Explain your reasoning.

a) $(x^{-1}y^{-2})^{-3}$ b) $(2a^{-2}b^2)^{-2}$

c) $(4m^2n^3)^{-3}$ d) $\left(\frac{3}{2}m^{-2}n^{-3}\right)^{-4}$

12. A cone with equal height and radius has volume 1234 cm^3. What is the height of the cone to the nearest tenth of a centimetre?

13. A sphere has volume 375 cubic feet. What is the surface area of the sphere to the nearest square foot?

14. Simplify. Which exponent laws did you use?

a) $\frac{(a^2b^{-1})^{-2}}{(a^{-3}b)^3}$ b) $\left(\frac{(c^{-3}d)^{-1}}{c^2d}\right)^{-2}$

15. Evaluate each expression for $a = -2$ and $b = 1$. Explain your strategy.

a) $(a^3b^2)(a^2b^3)$ b) $(a^{-1}b^{-2})(a^{-2}b^{-3})$

c) $\frac{a^{-4}b^5}{ab^3}$ d) $\left(\frac{a^{-7}b^7}{a^{-9}b^{10}}\right)^{-5}$

16. Simplify.

a) $m^{\frac{2}{3}} \cdot m^{\frac{4}{3}}$ b) $x^{-\frac{3}{2}} \div x^{-\frac{1}{4}}$

c) $\frac{-9a^{-4}b^{\frac{3}{4}}}{3a^2b^{\frac{1}{4}}}$ d) $\left(\frac{-64c^6}{a^9b^{-\frac{1}{2}}}\right)^{\frac{1}{3}}$

17. Identify any errors in each solution for simplifying an expression. Write a correct solution.

a) $(x^2y^{-3})(x^{\frac{1}{2}}y^{-1}) = x^2 \cdot x^{\frac{1}{2}} \cdot y^{-3} \cdot y^{-1}$
$= x^1 \cdot y^3$
$= xy^3$

b) $\left(\frac{-5a^2}{b^{\frac{1}{2}}}\right)^{-2} = \frac{10a^{-4}}{b^{-1}}$
$= \frac{10b}{a^4}$

18. Explain how to use a measuring cylinder containing water to calculate the diameter of a marble that fits inside the cylinder.

19. Identify the errors in each simplification. Write the correct solution.

a) $\frac{(m^{-3} \cdot n^2)^{-4}}{(m^2 \cdot n^{-3})^2} = (m^{-5} \cdot n^5)^{-6}$
$= m^{30} \cdot n^{30}$
$= (mn)^{30}$

b) $\left(r^{\frac{1}{2}} \cdot s^{-\frac{3}{2}}\right)^{\frac{1}{2}} \cdot \left(r^{-\frac{1}{4}} \cdot s^{\frac{1}{2}}\right)^{-1} = r^1 \cdot s^{-1} \cdot r^{-\frac{5}{4}} \cdot s^{-\frac{1}{2}}$
$= r^{\frac{1-5}{4}} \cdot s^{-1-\frac{1}{2}}$
$= r^{-\frac{1}{4}} \cdot s^{-\frac{3}{2}}$
$= \frac{1}{r^{\frac{1}{4}} \cdot s^{\frac{3}{2}}}$

2. a) i) $\sqrt[3]{35^2}$, or $\left(\sqrt[3]{35}\right)^2$

ii) $\sqrt{32^3}$, or $\left(\sqrt{32}\right)^3$

iii) $\sqrt[5]{(-32)^2}$, or $\left(\sqrt[5]{-32}\right)^2$

iv) $\sqrt{400^3}$, or $\left(\sqrt{400}\right)^3$

v) $\sqrt[3]{-125}$

vi) $\sqrt[3]{\left(\frac{8}{125}\right)^2}$, or $\left(\sqrt[3]{\frac{8}{125}}\right)^2$

b) iii) 4 iv) 8000

v) −5 vi) $\frac{4}{25}$

3. a) $4^{\frac{1}{3}}$

b) $9^{\frac{1}{2}}$, or $9^{0.5}$

c) $18^{\frac{1}{4}}$, or $18^{0.25}$

d) $10^{\frac{3}{2}}$, or $10^{1.5}$

e) $(-10)^{\frac{2}{3}}$

4. Approximately 53 s

5. $\sqrt[3]{3}$, $3^{\frac{2}{3}}$, $\left(\sqrt[3]{3}\right)^4$, $3^{\frac{3}{2}}$, $\left(\sqrt{3}\right)^5$

6. $\sqrt[3]{421\,875}$ mm, $421\,875^{\frac{1}{3}}$ mm, 75 mm

7. a) $\frac{81}{16}$ **b)** 4

c) $\frac{1}{100}$ **d)** 2

e) 100 **f)** 625

8. $4589.06

4.6 Applying the Exponent Laws, page 241

3. a) x^7 **b)** $\frac{1}{a^3}$

c) b^2 **d)** $\frac{1}{m}$

4. a) 0.5^5 **b)** 0.5^{-1}

c) 0.5^{-1} **d)** 0.5^5

5. a) x^2 **b)** $\frac{1}{x^3}$

c) n **d)** $\frac{1}{a^4}$

6. a) n^6 **b)** $\frac{1}{z^6}$

c) n^{12} **d)** $\frac{1}{c^4}$

7. a) $\left(\frac{3}{5}\right)^{12}$ **b)** $\left(\frac{3}{5}\right)^{-12}$

c) $\left(\frac{3}{5}\right)^{12}$ **d)** $\left(-\frac{3}{5}\right)^{12}$

8. a) $\frac{a^2}{b^2}$ **b)** $\frac{n^6}{m^3}$

c) $\frac{d^8}{c^8}$ **d)** $\frac{4b^2}{25c^2}$

e) a^2b^2 **f)** n^6m^3

g) $\frac{1}{c^{12}d^8}$ **h)** $\frac{x^3}{y^3}$

9. a) x; product of powers law

b) a^{-5}; product of powers law

c) b^3; product of powers law

d) 1; product of powers law

e) $\frac{1}{x^7}$; quotient of powers law

f) s^{10}; quotient of powers law

g) $\frac{1}{b^5}$; quotient of powers law

h) 1; quotient of powers law

10. a) 2.25 **b)** $\frac{9}{16}$

c) 0.36 **d)** 1

e) $\frac{5}{3}$ **f)** $-\frac{3}{8}$

g) $\frac{1000}{343}$ **h)** $\frac{3}{10}$

11. a) x^3y^6 **b)** $\frac{a^4}{4b^4}$

c) $\frac{1}{64m^6n^9}$ **d)** $\frac{16m^8n^{12}}{81}$

12. 10.6 cm

13. 251 ft.^{2}

14. a) $\frac{a^5}{b}$ **b)** $\frac{d^4}{c^2}$

15. a) −32 **b)** $-\frac{1}{8}$

c) $-\frac{1}{32}$ **d)** $\frac{1}{1024}$

16. a) m^2 **b)** $\frac{1}{x^{\frac{5}{4}}}$

c) $-\frac{3b^{\frac{1}{2}}}{a^6}$ **d)** $-\frac{4c^2b^{\frac{1}{6}}}{a^3}$

17. a) $\frac{x^{\frac{5}{2}}}{y^4}$ **b)** $\frac{b}{25a^4}$

19. a) $\frac{m^8}{n^{\frac{1}{2}}}$ **b)** $\frac{r^{\frac{1}{2}}}{s^{\frac{5}{4}}}$

20. a) **i)** Dimensions, in millimetres: $\frac{1000}{2^{\frac{7}{4}}}$ by $\frac{1000}{2^{\frac{5}{4}}}$;

297 mm by 420 mm

ii) Dimensions, in millimetres: $\frac{1000}{2^{\frac{9}{4}}}$ by $\frac{1000}{2^{\frac{7}{4}}}$;

210 mm by 297 mm

iii) Dimensions, in millimetres: $\frac{1000}{2^{\frac{11}{4}}}$ by $\frac{1000}{2^{\frac{9}{4}}}$;

149 mm by 210 mm

b) **i)** Dimensions, in millimetres: $\frac{1000}{2^{\frac{9}{4}}}$ by $\frac{1000}{2^{\frac{7}{4}}}$

ii) Dimensions, in millimetres: $\frac{1000}{2^{\frac{11}{4}}}$ by $\frac{1000}{2^{\frac{9}{4}}}$

iii) Dimensions, in millimetres: $\frac{1000}{2^{\frac{13}{4}}}$ by $\frac{1000}{2^{\frac{11}{4}}}$

c) A piece of A4 paper has the same dimensions as a folded piece of A3 paper; a piece of A5 paper has the same dimensions as a folded piece of A4 paper.

21. a) $\frac{a^{16}c^3}{b^7}$ **b)** $\frac{c^{14}}{64a^2b^{10}}$

22. a) $\frac{1}{a^{\frac{10}{9}}}$ **b)** $\frac{1}{a^{\frac{7}{2}}}$

23. For example:

a) $x^1 \cdot x^{\frac{1}{2}}$, $x^{\frac{3}{4}} \cdot x^{\frac{3}{4}}$, $x^2 \cdot x^{-\frac{1}{2}}$

b) $x^2 \div x^{\frac{1}{2}}$, $x^{\frac{5}{2}} \div x^1$, $x^{-1} \div x^{-\frac{5}{2}}$

c) $\left(x^{\frac{1}{2}}\right)^3$, $\left(x^6\right)^{\frac{1}{4}}$, $\left(x^{-\frac{1}{3}}\right)^{-\frac{9}{2}}$

24. $\frac{1}{2}\left(\frac{3}{2}\right)^{\frac{1}{2}}$ cm, or approximately 0.6 cm

Chapter 4: Review, page 246

1. a) 10 **b)** 0.9

c) 2 **d)** $\frac{3}{5}$

2. The index tells which root to take.

3. a) 3.3 **b)** –2.3

c) 2.0

4. a) 25 **b)** 216

c) 2401

5. Neither

6. a) Rational **b)** Rational

c) Rational **d)** Irrational

e) Rational **f)** Rational

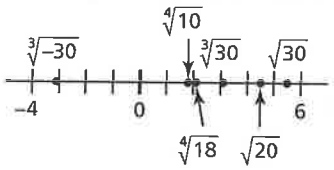
g) Rational **h)** Irrational

i) Irrational

7. Approximately 4.8 cm

8. a) Rational **b)** Irrational

9. $\sqrt[3]{-30}$, $\sqrt[4]{10}$, $\sqrt[4]{18}$, $\sqrt[3]{30}$, $\sqrt{20}$, $\sqrt{30}$



10. 1 s

11. a) $5\sqrt{6}$ **b)** $3\sqrt[3]{5}$

c) $4\sqrt{7}$ **d)** $3\sqrt[4]{2}$

12. a) $\sqrt{180}$ **b)** $\sqrt{126}$

c) $\sqrt[3]{192}$ **d)** $\sqrt[4]{32}$

13. Approximately 1.0 cm

15. $6\sqrt{2}$, $3\sqrt{6}$, $5\sqrt{2}$, $4\sqrt{3}$, $2\sqrt{7}$

17. a) $\sqrt[4]{12}$ **b)** $\sqrt[3]{(-50)^5}$, or $\left(\sqrt[3]{-50}\right)^5$

c) $\sqrt{1.2}$ **d)** $\sqrt[3]{\frac{3}{8}}$

18. a) $1.4^{\frac{1}{2}}$ **b)** $13^{\frac{2}{3}}$

c) $2.5^{\frac{4}{5}}$ **d)** $\left(\frac{2}{5}\right)^{\frac{3}{4}}$

19. a) 2 **b)** 1.2

c) –32 **d)** $\frac{27}{64}$

20. Approximately 35%

21. $\left(\sqrt{5}\right)^3$, $5^{\frac{3}{4}}$, $5^{\frac{2}{3}}$, $\sqrt[3]{5}$, $\sqrt[4]{5}$