ARITHMETIC SERIES

ii) $-5 + 1 + 7 + 13 + \ldots + T_{12}$

we know $T_1 = -5$

$T_n = T_{12} = ?$

$n = 12$

$d = +6$

$$S_n = \frac{n}{2}\left[T_1 + T_n\right]$$

we don't know the value of $T_n = T_{12}$, so we must use our other formula

$$T_n = T_1 + (n-1)d$$

we know all the terms to solve

$$T_n = T_{12}$$

$$T_n = -5 + (12-1)(6)$$

$$T_n = -5 + 11(6)$$

$$\boxed{T_n = +61}$$

Now we can solve S_n

$$S_n = \frac{n}{2}\left[T_1 + T_n\right] \qquad n = 12$$

$$S_{12} = \frac{12}{2}\left[-5 + 61\right] = 6\left[56\right]$$

$$\boxed{S_{12} = 336}$$

vi) $6 + \frac{9}{2} + 3 + \frac{3}{2} + \ldots - 28.5$

We know $T_1 = 6$

$T_n = -28.5$

$n = ?$

$d = \frac{9}{2} - 6 = \frac{9}{2} - \frac{12}{2}$

$d = \frac{-3}{2} = -1.5$

$S_n = \frac{n}{2}\left[T_1 + T_n\right]$ but we don't know how many terms "n"

Check our other formula

$T_n = T_1 + (n-1)d$ we can solve for n.

$T_n - T_1 = T_1 - T_1 + (n-1)d$

$\frac{T_n - T_1}{d} = (n-1)\frac{d}{d}$

$\frac{T_n - T_1}{d} + 1 = n - 1 + 1$

$n = \frac{T_n - T_1}{d} + 1$

$n = \frac{-28.5 - 6}{-1.5} + 1 \rightarrow \boxed{n = 24}$

Can now solve for S_n

$S_n = \frac{24}{2}\left[6 + (-28.5)\right]$

$\boxed{S_n = -270}$

2a) $T_1 = a = 12$

$n = 20$

$d = 4$

$S_{20} = ?$

$T_n = T_{20} = ?$

$$S_{20} = \frac{n}{2}\left[T_1 + T_n\right]$$ Must first find T_n

$$T_n = T_1 + (n-1)d$$

$$T_{20} = 12 + (20-1)(4)$$

$$T_{20} = 12 + 76$$

$$T_{20} = 88$$

$$S_n = S_{20}$$

$$S_{20} = \frac{n}{2}\left[T_1 + T_{20}\right]$$

$$S_{20} = \frac{20}{2}\left[12 + 88\right]$$

$$S_{20} = 10\left[100\right]$$

$$\boxed{S_{20} = 1000}$$

2d) $d = -7$

$n = 16$

$S_n = -600$

$T_1 = a = ?$

$$S_n = \frac{n}{2}\left[T_1 + T_n\right]$$

We don't know T_1 OR T_n.

We have 2 unknowns

$$-600 = \frac{16}{2}\left[T_1 + T_n\right]$$

$$\frac{-600}{8} = \frac{8}{8}\left[T_1 + T_n\right]$$

$$\boxed{-75 = T_1 + T_n}$$

Need a second equation to solve T_1 & T_n.

$$T_n = T_1 + (n-1)d$$

$$T_n = T_1 + (16-1)(-7)$$

$$\boxed{T_n = T_1 - 105}$$

Now use substitution

$$-75 = T_1 + T_n$$

$$-75 = T_1 + T_1 - 105$$

$$-75 \overset{+105}{} = 2T_1 - 105 \overset{+105}{}$$

$$\frac{30}{2} = \frac{2T_1}{2}$$

$$\boxed{T_1 = 15}$$

$$T_n = T_1 - 105$$

$$T_n = 15 - 105$$

$$\boxed{T_n = T_{16} = -90}$$

3b) $T_n = 7 + 8n \qquad S_9 = ?$

$T_9 = 7 + 8(9) \qquad T_1 = 7 + 8(1)$

$T_9 = 7 + 72 \qquad T_1 = 15$

$T_9 = 79$

$$S_9 = \frac{n}{2}\left[T_1 + T_n\right]$$

$$S_9 = \frac{9}{2}\left[15 + 79\right]$$

$$\boxed{S_9 = 423}$$