

Discuss the Ideas

- When m is an integer, describe the relationship between a^m and a^{-m} .
- Why is there usually more than one way to determine the value of a power of the form $a^{\frac{m}{n}}$? Use examples to justify your answer.

Exercises

A

3. Copy then complete each equation.

a) $\frac{1}{5^4} = 5^{\square}$ b) $\left(-\frac{1}{2}\right)^{-3} = (-2)^{\square}$

c) $\frac{1}{3^{\square}} = 3^2$ d) $\frac{1}{4^{-2}} = 4^{\square}$

4. Evaluate the powers in each pair without a calculator.

a) 4^2 and 4^{-2} b) 2^4 and 2^{-4}

c) 6^1 and 6^{-1} d) 4^3 and 4^{-3}

Describe what is similar about the answers, and what is different.

5. Given that $2^{10} = 1024$, what is 2^{-10} ?

6. Write each power with a positive exponent.

a) 2^{-3} b) 3^{-5} c) $(-7)^{-2}$

7. Write each power with a positive exponent.

a) $\left(\frac{1}{2}\right)^{-2}$ b) $\left(\frac{2}{3}\right)^{-3}$ c) $\left(-\frac{6}{5}\right)^{-4}$

8. Evaluate each power without using a calculator.

a) 3^{-2} b) 2^{-4} c) $(-2)^{-5}$

d) $\left(\frac{1}{3}\right)^{-3}$ e) $\left(-\frac{2}{3}\right)^{-2}$ f) $\frac{1}{5^{-3}}$

B

9. Evaluate each power without using a calculator.

a) $4^{-\frac{1}{2}}$ b) $0.09^{-\frac{1}{2}}$

c) $27^{-\frac{1}{3}}$ d) $(-64)^{-\frac{1}{3}}$

e) $(-0.027)^{-\frac{2}{3}}$ f) $32^{-\frac{2}{5}}$

g) $9^{-\frac{3}{2}}$ h) $0.04^{-\frac{3}{2}}$

10. Use a power with a negative exponent to write an equivalent form for each number.

a) $\frac{1}{9}$ b) $\frac{1}{5}$ c) 4 d) -3

11. When you save money in a bank, the bank pays you *interest*. This interest is added to your investment and the resulting amount also earns interest. We say the interest *compounds*. Suppose you want an amount of \$3000 in 5 years. The interest rate for the savings account is 2.5% compounded annually. The money, P dollars, you must invest now is given by the formula: $P = 3000(1.025)^{-5}$. How much must you invest now to have \$3000 in 5 years?

12. Here is a student's solution for evaluating a power. Identify any errors in the solution. Write a correct solution.

$$\begin{aligned} \left(-\frac{64}{125}\right)^{-\frac{5}{3}} &= \left(\frac{64}{125}\right)^{\frac{5}{3}} \\ &= \left(\sqrt[3]{\frac{64}{125}}\right)^5 \\ &= \left(\frac{4}{5}\right)^5 \\ &= \frac{1024}{3125} \end{aligned}$$

13. Evaluate each power without using a calculator.

a) $27^{-\frac{4}{3}}$ b) $16^{-1.5}$ c) $32^{-0.4}$

d) $\left(-\frac{8}{27}\right)^{-\frac{2}{3}}$ e) $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$ f) $\left(\frac{9}{4}\right)^{-\frac{5}{2}}$

14. Michelle wants to invest enough money on January 1st to pay her nephew \$150 at the end of each year for the next 10 years. The savings account pays 3.2% compounded annually. The money, P dollars, that Michelle must invest today is given by the formula $P = \frac{150[1 - 1.032^{-10}]}{0.032}$. How much must Michelle invest on January 1st?

20. a) Approximately 93%
 b) Approximately 81%
 c) 5 h
21. Mars; period of Earth: approximately 363.8 Earth days;
 period of Mars: approximately 688.5 Earth days
22. Karen

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3. a) $\frac{1}{5^4} = 5^{-4}$ b) $\left(-\frac{1}{2}\right)^{-3} = (-2)^3$
 c) $\frac{1}{3^{-2}} = 3^2$ d) $\frac{1}{4^{-2}} = 4^2$
4. a) $16, \frac{1}{16}$ b) $16, \frac{1}{16}$
 c) $6, \frac{1}{6}$ d) $64, \frac{1}{64}$
5. $\frac{1}{1024}$
6. a) $\frac{1}{2^3}$
 b) $\frac{1}{3^5}$
 c) $\frac{1}{(-7)^2}$, or $\frac{1}{7^2}$
7. a) 2^2
 b) $\left(\frac{3}{2}\right)^3$
 c) $\left(-\frac{5}{6}\right)^4$, or $\left(\frac{5}{6}\right)^4$
8. a) $\frac{1}{9}$ b) $\frac{1}{16}$
 c) $-\frac{1}{32}$ d) 27
 e) $\frac{9}{4}$ f) 125
9. a) $\frac{1}{2}$ b) $\frac{10}{3}$
 c) $\frac{1}{3}$ d) $-\frac{1}{4}$
 e) $\frac{100}{9}$ f) $\frac{1}{4}$
 g) $\frac{1}{27}$ h) 125

10. Answers may vary. For example:

- a) 3^{-2}
 b) $25^{-\frac{1}{2}}$
 c) $\left(\frac{1}{2}\right)^{-2}$
 d) $\left(\frac{1}{-27}\right)^{-\frac{1}{3}}$

11. \$2651.56

12. $\frac{3125}{1024}$

13. a) $\frac{1}{81}$ b) $\frac{1}{64}$
 c) $\frac{1}{4}$ d) $\frac{9}{4}$
 e) $\frac{8}{27}$ f) $\frac{32}{243}$

14. \$1266.57

15. Approximately 0.19%

16. 5^{-2} ; $\frac{1}{25} > \frac{1}{32}$

17. a) The numbers at the left are divided by 2 each time. The exponents in the powers at the right decrease by 1 each time.

b) $2 = 2^1$; $1 = 2^0$; $\frac{1}{2} = 2^{-1}$; $\frac{1}{4} = 2^{-2}$; $\frac{1}{8} = 2^{-3}$

18. 3^8 , or 6561 times as great

19. a) The exponent is positive.

b) The exponent is negative.

c) The exponent is 0.

20. No; if the base is between 0 and 1, the power will be

greater than 1. For example: $\left(\frac{1}{2}\right)^{-1} = 2$

21. a) Approximately 2.0×10^{20} N

b) Answers may vary depending on researched values.

For example: approximately 1.9×10^{20} N

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1. a) 2 b) 7
 c) 16 d) $\frac{343}{27}$
 e) -32