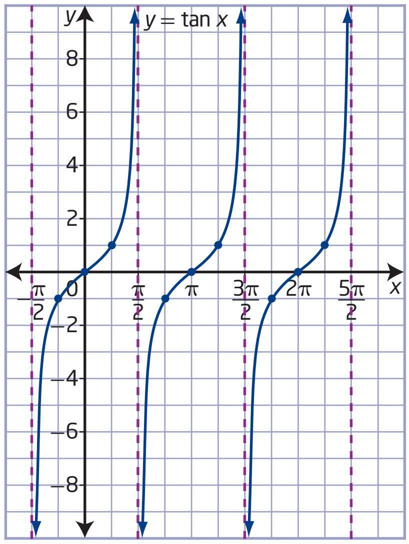
Key Ideas

- You can use asymptotes and three points to sketch one cycle of a tangent function. To graph $y = \tan x$, draw one asymptote; draw the points where $y = -1$, $y = 0$, and $y = 1$; and then draw another asymptote.

How can you determine the location of the asymptotes for the function $y = \tan x$?



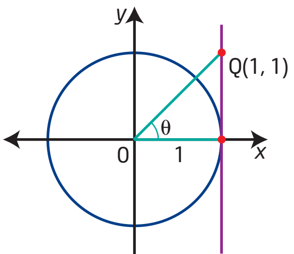
- The tangent function $y = \tan x$ has the following characteristics:
  - The period is π.
  - The graph has no maximum or minimum values.
  - The range is $\{y \mid y \in \mathrm{R}\}$.
  - Vertical asymptotes occur at $x = \frac{\pi}{2} + n\pi, n \in \mathrm{I}$.
  - The domain is $\left\{x \mid x \neq \frac{\pi}{2} + n\pi, x \in \mathrm{R}, n \in \mathrm{I}\right\}$.
  - The x-intercepts occur at $x = n\pi, n \in \mathrm{I}$.
  - The y-intercept is 0.

Check Your Understanding

Practise

1. For each diagram, determine $\tan \theta$ and the value of θ, in degrees. Express your answer to the nearest tenth, when necessary.

a)



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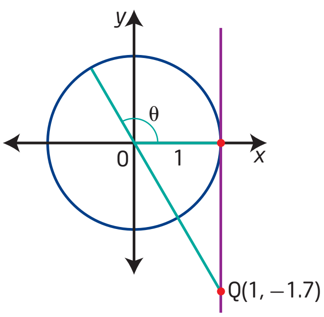

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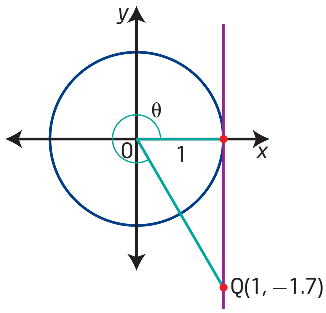

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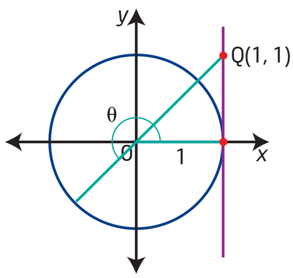
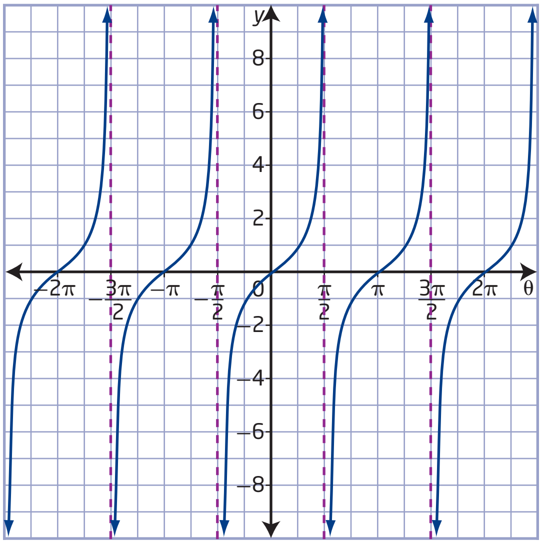


2. Use the graph of the function $y = \tan \theta$ to determine each value.



a) $\tan \frac{\pi}{2}$

b) $\tan \frac{3\pi}{4}$

c) $\tan \left(-\frac{7\pi}{4}\right)$

d) $\tan 0$

e) $\tan \pi$

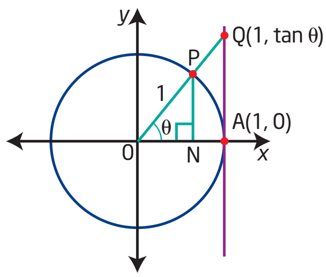
f) $\tan \frac{5\pi}{4}$

3. Does $y = \tan x$ have an amplitude? Explain.

4. Use graphing technology to graph $y = \tan x$ using the following window settings: x: [−360°, 360°, 30°] and y: [−3, 3, 1]. Trace along the graph to locate the value of $\tan x$ when $x = 60°$. Predict the other values of x that will produce the same value for $\tan x$ within the given domain. Verify your predictions.

Apply

5. In the diagram, △PON and △QOA are similar triangles. Use the diagram to justify the statement $\tan \theta = \frac{\sin \theta}{\cos \theta}$.



6. Point P(x, y) is plotted where the terminal arm of angle θ intersects the unit circle.

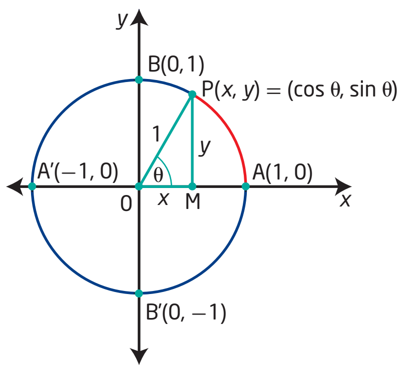
a) Use P(x, y) to determine the slope of the terminal arm.

b) Explain how your result from part a) is related to tan θ.

c) Write your results for the slope from part a) in terms of sine and cosine.

d) From your answer in part c), explain how you could determine tan θ when the coordinates of point P are known.

7. Consider the unit circle shown.



a) From △POM, write the ratio for tan θ.

b) Use cos θ and sin θ to write the ratio for tan θ.

c) Explain how your answers from parts a) and b) are related.

8. The graph of $y = \tan \theta$ appears to be vertical as θ approaches 90°.

a) Copy and complete the table. Use a calculator to record the tangent values as θ approaches 90°.

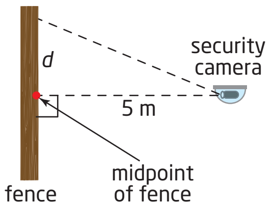
θ	tan θ
89.5°	
89.9°	
89.999°	
89.999 999°	

b) What happens to the value of tan θ as θ approaches 90°?

c) Predict what will happen as θ approaches 90° from the other direction.

θ	tan θ
90.5°	
90.01°	
90.001°	
90.000 001°	

9. A security camera scans a long straight fence that encloses a section of a military base. The camera is mounted on a post that is located 5 m from the midpoint of the fence. The camera makes one complete rotation in 60 s.



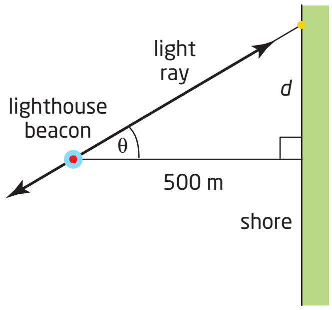
a) Determine the tangent function that represents the distance, d, in metres, along the fence from its midpoint as a function of time, t, in seconds, if the camera is aimed at the midpoint of the fence at $t = 0$.

b) Graph the function in the interval $-15 \leq t \leq 15$.

c) What is the distance from the midpoint of the fence at $t = 10$ s, to the nearest tenth of a metre?

d) Describe what happens when $t = 15$ s.

10. A rotating light on top of a lighthouse sends out rays of light in opposite directions. As the beacon rotates, the ray at angle θ makes a spot of light that moves along the shore. The lighthouse is located 500 m from the shoreline and makes one complete rotation every 2 min.



a) Determine the equation that expresses the distance, d, in metres, as a function of time, t, in minutes.

b) Graph the function in part a).

c) Explain the significance of the asymptote in the graph at θ = 90°.

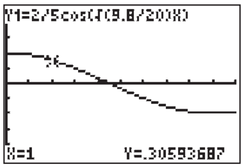
Did You Know?

The Fisgard Lighthouse was the first lighthouse built on Canada's west coast. It was built in 1860 before Vancouver Island became part of Canada and is located at the entrance to Esquimalt harbour.

27. a) $y = 3\sin(x + \pi) + 2$ **b)** $y = 3\sin 2\left(x - \frac{\pi}{2}\right) + 1$

c) $y = 2\sin\left(x + \frac{\pi}{2}\right) + 5$ **d)** $y = 5\sin 3(x - 120°) - 1$

28. a) $P = \frac{2}{5}\cos\sqrt{\frac{9.8}{20}}t$



b) approximately −0.20 radians or 3.9 cm along the arc to the left of the vertical

C1 a changes the amplitude, b changes the period, c changes the phase shift, d changes the vertical translation; Answers may vary.

C2 a) They are exactly same.
b) This is because the sine of a negative number is the same as the negative sine of the number.
c) They are mirror images reflected in the x-axis.
d) It is correct.

C3 $\frac{5\pi}{4}$ square units

C4 a) $0 < b < 1$ **b)** $a > 1$
c) Example: $c = 0$, $d = 0$ **d)** $d > a$
e) Example: $c = -\frac{\pi}{2}$, $b = 1$, $d = 0$ **f)** $b = 3$

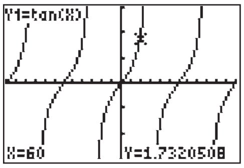
5.3 The Tangent Function, pages 262 to 265

1. a) 1, 45° **b)** −1.7, 120.5°
c) −1.7, 300.5° **d)** 1, 225°

2. a) undefined **b)** −1 **c)** 1
d) 0 **e)** 0 **f)** 1

3. No. The tangent function has no maximum or minimum, so there is no amplitude.

4.



−300°, −120°, 240°

5. $\frac{\tan\theta}{\sin\theta} = \frac{1}{\cos\theta}$; $\tan\theta = \frac{\sin\theta}{\cos\theta}$

6. a) slope $= \frac{y}{x}$
b) Since y is equal to $\sin\theta$ and x is equal to $\cos\theta$, then $\tan\theta = \frac{y}{x}$.
c) slope $= \frac{\sin\theta}{\cos\theta}$ **d)** $\tan\theta = \frac{y}{x}$

7. a) $\tan\theta = \frac{y}{x}$ **b)** $\tan\theta = \frac{\sin\theta}{\cos\theta}$
c) $\sin\theta$ and $\cos\theta$ are equal to y and x, respectively.

8. a)

θ	tan θ
89.5°	114.59
89.9°	572.96
89.999°	57 295.78
89.999 999°	57 295 779.51

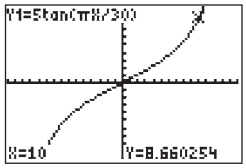
b) The value of tan θ increases to infinity.

c)

θ	tan θ
90.5°	−114.59
90.01°	−5729.58
90.001°	−57 295.78
90.000 001°	−57 295 779.51

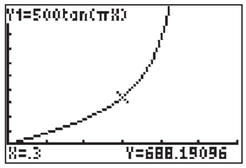
The value of tan θ approaches negative infinity.

9. a) $d = 5\tan\frac{\pi}{30}t$
b)



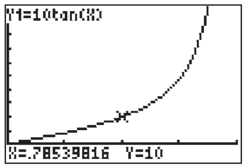
c) 8.7 m
d) At $t = 15$ s, the camera is pointing along a line parallel to the wall and is turning away from the wall.

10. a) $d = 500\tan\pi t$
b)



c) The asymptote represents the moment when the ray of light shines along a line that is parallel to the shore.

11. $d = 10\tan x$



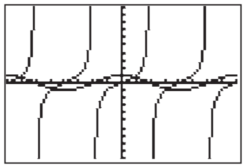
12. a) a tangent function
b) The slope would be undefined. It represents the place on the graph where the asymptote is.

13. Example:
a) (4, 3) **b)** 0.75
c) tan θ is the slope of the graph.

14. a) $\tan 0.5 \approx 0.5463$, power series ≈ 0.5463
b) $\sin 0.5 \approx 0.4794$, power series ≈ 0.4794
c) $\cos 0.5 \approx 0.8776$, power series ≈ 0.8776

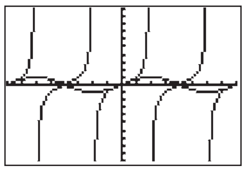
C1 The domain of $y = \sin x$ and $y = \cos x$ is all real numbers. The tangent function is not defined at $x = \frac{\pi}{2} + n\pi$, $n \in I$. Thus, these numbers must be excluded from the domain of $y = \tan x$.

C2 a)



Example: The tangent function has asymptotes at the same x-values where zeros occur on the cosine function.

b)



Example: The tangent function has zeros at the same x-values where zeros occur on the sine function.

C3 Example: A circular or periodic function repeats its values over a specific period. In the case of $y = \tan x$, the period is π. So, the equation $\tan(x + \pi) = \tan x$ is true for all x in the domain of tan x.

5.4 Equations and Graphs of Trigonometric Functions, pages 275 to 281

1. a) $x = 0, \pi, 2\pi$ **b)** $x = \pi n$ where n is an integer
c) $x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi$

2. Examples:
a) 1.25, 4.5
b) −3, −1.9, 0.1, 1.2, 3.2, 4.1, 6.3, 7.2

3. Examples: −50°, −10°, 130°, 170°, 310°, 350°