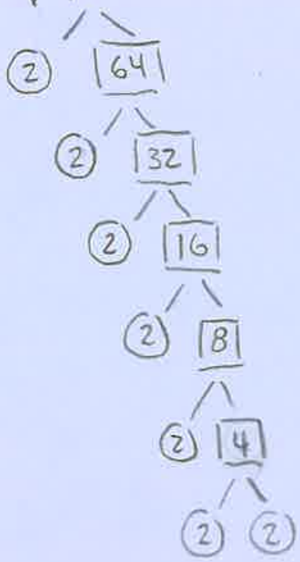
① b) $\sqrt{128}$



$$= \sqrt{2\times2\times2\times2\times2\times2\times2}$$

$$= \sqrt{2\times2}\cdot\sqrt{2\times2}\cdot\sqrt{2\times2}\cdot\sqrt{2}$$

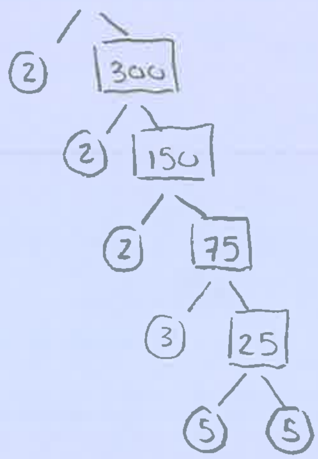
$$= \sqrt{4}\cdot\sqrt{4}\cdot\sqrt{4}\cdot\sqrt{2}$$

$$= 2\cdot2\cdot2\cdot\sqrt{2}$$

$$= \boxed{8\sqrt{2}}$$

Looking for perfect squares (2 of the same number)

d) $\sqrt{600}$



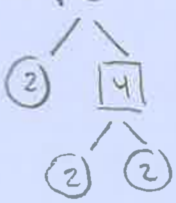
$$= \sqrt{2\times2\times2\times3\times5\times5}$$

$$= \sqrt{2\times2}\cdot\sqrt{5\times5}\times\sqrt{2}\times\sqrt{3}$$

$$= 2\cdot5\cdot\sqrt{2\cdot3}$$

$$= \boxed{10\sqrt{6}}$$

e) $3\sqrt{8}$



$$= 3\sqrt{2\times2\times2}$$

$$= 3\cdot\sqrt{2\times2}\cdot\sqrt{2}$$

$$= 3\cdot2\cdot\sqrt{2}$$

$$= \boxed{6\sqrt{2}}$$

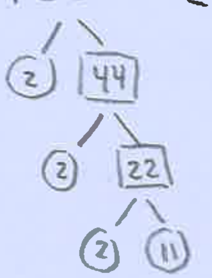
f) $\sqrt{a^3b^4c} = \sqrt{a\cdot a\cdot a\cdot b\cdot b\cdot b\cdot b\cdot c}$

$= \sqrt{a\cdot a}\cdot\sqrt{b\cdot b}\cdot\sqrt{b\cdot b}\cdot\sqrt{a\cdot c}$

$= a\cdot b\cdot b\cdot\sqrt{ac}$

$= \boxed{ab^2\sqrt{ac}}$

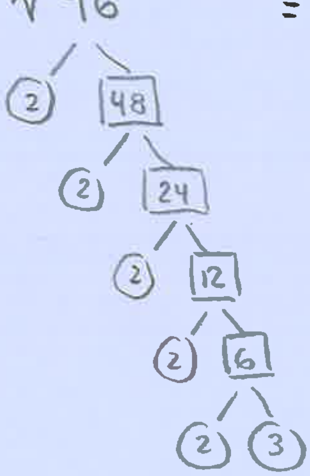
g) $\sqrt[3]{88} = \sqrt[3]{2\cdot 2\cdot 2\cdot 11}$



$= \boxed{2\sqrt[3]{11}}$

Cube Root requires perfect cubes (3 of same number)

i) $\sqrt[4]{96} = \sqrt[4]{2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 3}$



$= \boxed{2\sqrt[4]{6}}$

Quad (4) root means we are looking for 4 of the same number

j) $\sqrt[3]{a^5b^3c^4} = \sqrt[3]{a\cdot a\cdot a\cdot a\cdot a\cdot b\cdot b\cdot b\cdot c\cdot c\cdot c\cdot c}$

$= \boxed{abc\sqrt[3]{a^2c}}$

1) $\sqrt[n]{a^n b^{n+2} c^{n-1}} = abc\sqrt[n]{a^{n-n} b^{n+2-n} c^{n-1-n}}$

$= abc\sqrt[n]{a^0 b^2 c^{-1}}$

$= \boxed{abc \cdot \sqrt[n]{\frac{b^2}{c}}}$

2)

a) $5\sqrt{9} + 2\sqrt{49} = \underbrace{5\sqrt{3\times3}}_{} + \underbrace{2\sqrt{7\times7}}_{}$

3 3 7 7

Perfect Squares

$= 5(3) + 2(7)$

$= 15 + 14$

$= \boxed{29}$

c) $2\sqrt{10} + 7\sqrt{10} - 6\sqrt{10}$ (can add because same radical $\sqrt{10}$)

$= \boxed{3\sqrt{10}}$

e) $5\sqrt{3} - 7\sqrt{12} + 2\sqrt{27}$

2 6 3 9

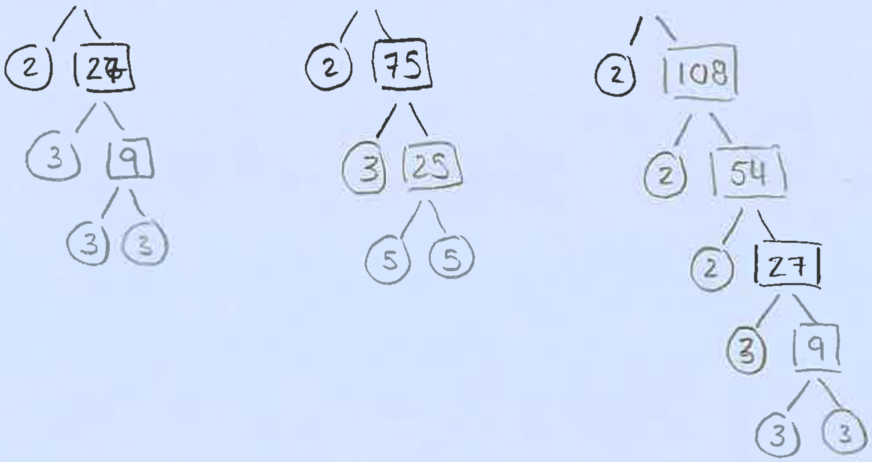
2 3 3 3

$= 5\sqrt{3} - 7\sqrt{2\times2\times3} + 2\sqrt{3\times3\times3}$

$= 5\sqrt{3} - 7(2)\sqrt{3} + 2(3)\sqrt{3}$

$= 5\sqrt{3} - 14\sqrt{3} + 6\sqrt{3} = \boxed{15\sqrt{3}}$

g) $\sqrt{54} + \sqrt{150} - 2\sqrt{216}$

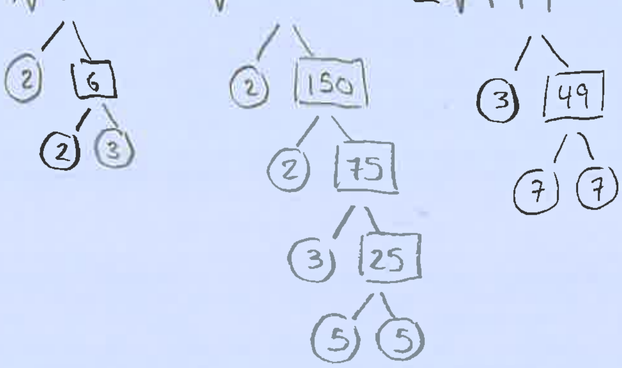


$= \sqrt{3\cdot3\cdot3\cdot2} + \sqrt{2\cdot3\cdot5\cdot5} - 2\sqrt{2\cdot2\cdot2\cdot3\cdot3\cdot3}$

$= 3\sqrt{6} + 5\sqrt{6} - 12\sqrt{6}$

$= \boxed{-4\sqrt{6}}$

h) $4\sqrt{12} + \sqrt{300} - 2\sqrt{147}$



$= 4\sqrt{2\cdot2\cdot3} + \sqrt{2\cdot2\cdot3\cdot5\cdot5} - 2\sqrt{7\cdot7\cdot3}$

$= 8\sqrt{3} + 10\sqrt{3} - 14\sqrt{3}$

$= \boxed{4\sqrt{3}}$

c) $b\sqrt{27a^3b} - a\sqrt{3ab^3} - 2\sqrt{75a^3b^3} + 4\sqrt[3]{a^4b^4}$

$$b\sqrt{27a^3b} = b\sqrt{3\cdot 3\; 3\; a\cdot a\; a\cdot b} = 3ab\sqrt{3ab}$$

$$a\sqrt{3ab^3} = a\sqrt{3\cdot a\cdot b\cdot b\; b} = ab\sqrt{3ab}$$

$$2\sqrt{75a^3b^3} = 2\sqrt{3\; 5\cdot 5\; a\cdot a\; a\; b\cdot b\cdot b} = 10ab\sqrt{3ab}$$

$$4\sqrt[3]{a^4b^4} = 4\sqrt[3]{a\cdot a\cdot a\cdot a\cdot b\cdot b\cdot b\; b} = \underbrace{4ab\sqrt[3]{ab}}_{\text{Note } \sqrt[3]{}}$$

$$3ab\sqrt{3ab} - ab\sqrt{3ab} + 10ab\sqrt{3ab} + 4ab\sqrt[3]{ab}$$

$$ab\sqrt{3ab}\,(3 - 1 + 10) + 4ab\sqrt[3]{ab}$$

$$\boxed{12ab\sqrt{3ab} + 4ab\sqrt[3]{ab}}$$

(4) a)

$-6\sqrt{2} = -\sqrt{6\cdot 6\cdot 2} = -\sqrt{72}$ 4

$-3\sqrt{7} = -\sqrt{3\cdot 3\cdot 7} = -\sqrt{63}$ 6

$-2\sqrt{17} = -\sqrt{2\cdot 2\cdot 17} = -\sqrt{68}$ 5

$-4\sqrt{5} = -\sqrt{4\cdot 4\cdot 5} = -\sqrt{80}$ 2

$-2\sqrt{21} = -\sqrt{2\cdot 2\cdot 21} = -\sqrt{84}$ 1

$-5\sqrt{3} = -\sqrt{5\cdot 5\cdot 3} = -\sqrt{75}$ 3

ORDER OF LEAST TO GREATEST

$-2\sqrt{21}, -4\sqrt{5}, -5\sqrt{3}, -6\sqrt{2}, -2\sqrt{17}, -3\sqrt{7}$

⑤ Perimeter $= 6\sqrt{6} + 6\sqrt{6} + \frac{4\sqrt{2}}{3} + \frac{4\sqrt{2}}{3}$

$$\boxed{P = 12\sqrt{6} + \frac{8\sqrt{2}}{3}}$$

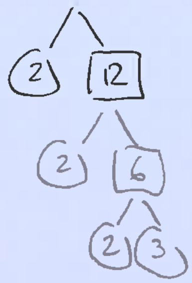
AREA $= 6\sqrt{6}\left(\frac{4\sqrt{2}}{3}\right) = \frac{24\sqrt{12}}{3}$

$A = \frac{24\sqrt{12}}{3} = \frac{24\sqrt{2\cdot 2\cdot 3}}{3} = \frac{48\sqrt{3}}{3}$

$$\boxed{A = 16\sqrt{3}}$$

① When Multiplying Radicals, we can SIMPLIFY RADICALS BEFORE <u>OR</u> AFTER Multiplication.

a) $\sqrt{24} \times \sqrt{6}$ We will simplify Before



$$\sqrt{24} \times \sqrt{6} = \sqrt{2 \times 2 \times 2 \times 3} \times \sqrt{2 \times 3}$$

$$= 2\sqrt{6} \cdot \sqrt{6}$$

$$= 2\sqrt{6 \cdot 6} = 2\sqrt{36}$$

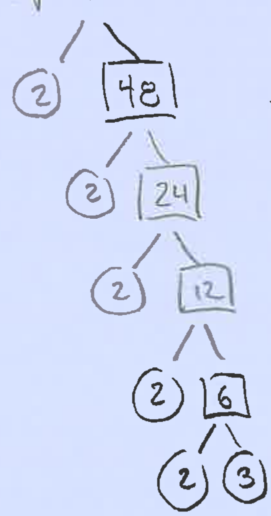
$$= 2(6)$$

$$= \boxed{12}$$

b) $3\sqrt{12} \cdot 5\sqrt{8}$ We will simplify After

$$3\sqrt{12} \cdot 5\sqrt{8} = 3(5) \cdot \sqrt{12 \cdot 8}$$

$$= 15\sqrt{96} = 15\sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}$$



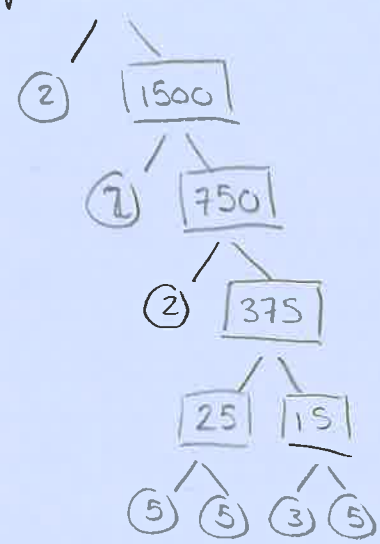
$$= 15 \cdot 2 \cdot 2 \cdot \sqrt{6}$$

$$= \boxed{60\sqrt{6}}$$

c) $5\sqrt[3]{50} \cdot 3\sqrt[3]{60}$ Going to simplify at the end

$$5\sqrt[3]{50} \cdot 3\sqrt[3]{60} = 5(3) \cdot \sqrt[3]{50 \cdot 60}$$

$$= 5(3) \cdot \sqrt[3]{3000}$$



$$= 5(3) \cdot \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 \cdot 3}$$

$$= 5(3)(2)(5)\sqrt[3]{3}$$

$$= \boxed{150\sqrt[3]{3}}$$

e) $\sqrt[3]{a^2bc^3} \cdot \sqrt[3]{a^5b^4c^2}$ Simplify AFTER

$$= \sqrt[3]{a^{2+5}\, b^{1+4}\, c^{3+2}}$$

$$= \sqrt[3]{a^7b^5c^5}$$ Can pull perfect Cubes

$$= \sqrt[3]{a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b \cdot b \cdot c \cdot c \cdot c \cdot c \cdot c}$$

$$= \boxed{a^2bc\sqrt[3]{ab^2c^2}}$$

f) $\sqrt[4]{32x^3y} \cdot \sqrt[4]{64x^2y^7}$ Going to SIMPLIFY FIRST

$\sqrt[4]{32x^3y} = \sqrt[4]{2\cdot2\cdot2\cdot2\cdot2\cdot x\cdot x\cdot x\cdot y} = 2\sqrt[4]{2x^3y}$

$\sqrt[4]{64x^2y^7} = \sqrt[4]{2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot x\cdot x\cdot y\cdot y\cdot y\cdot y\cdot y\cdot y\cdot y} = 2y\sqrt[4]{4x^2y^3}$

$2\sqrt[4]{2x^3y} \cdot 2y\sqrt[4]{4x^2y^3} = 4y\sqrt[4]{8x^5y^4}$

$= 4y\sqrt[4]{2\cdot2\cdot2\cdot x\cdot x\cdot x\cdot x\cdot x\cdot y\cdot y\cdot y\cdot y}$

$= \boxed{4y^2x\sqrt[4]{8x}}$

g) $2\sqrt{3}\left(4\sqrt{21} + 5\sqrt{15}\right)$

$2(4)\sqrt{3\cdot 21} + 2(5)\cdot\sqrt{3\cdot 15}$

$= 8\sqrt{3\cdot3\cdot7} + 10\sqrt{3\cdot3\cdot5}$

$= \boxed{24\sqrt{7} + 30\sqrt{5}}$

j) $\left(3\sqrt{2} + 4\sqrt{3}\right)\left(5\sqrt{3} - \sqrt{8}\right)$

$= 3(5)\cdot\sqrt{2\cdot 3} - 3\sqrt{2\cdot 8} + 4(5)\sqrt{3\cdot 3} - 4\sqrt{3\cdot 8}$

$= 15\sqrt{6} - 3\sqrt{2\cdot 2\cdot 2\cdot 2} + 20(3) - 4\sqrt{3\cdot 2\cdot 2\cdot 2}$

$= 15\sqrt{6} - 3(2)(2) + 60 - 4(2)\sqrt{6}$

$= 15\sqrt{6} - 12 + 60 - 8\sqrt{6}$

$= \boxed{7\sqrt{6} + 48}$

L) $\left(3\sqrt[3]{8x^2} + \sqrt[3]{4x^2}\right)\left(\sqrt[3]{2x^2} - 6\sqrt[3]{8x^2}\right)$

$= 3\sqrt[3]{8x^2\cdot 2x^2} - 6(3)\sqrt[3]{8x^2\cdot 8x^2} + \sqrt[3]{4x^2\cdot 2x^2} - 6\sqrt[3]{4x^2\cdot 8x^2}$

$= 3\sqrt[3]{16x^4} - 18\sqrt[3]{64x^4} + \sqrt[3]{8x^4} - 6\sqrt[3]{32x^4}$

$= 3\sqrt[3]{2\cdot 2\cdot 2\cdot 2\cdot x^4} - 18\sqrt[3]{2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot x^4} + \sqrt[3]{2\cdot 2\cdot 2\cdot x^4} - 6\sqrt[3]{2\cdot 2\cdot 2\cdot 2\cdot 2\cdot x^4}$

$= 6x\sqrt[3]{2x} - 72x\sqrt[3]{x} + 2x\sqrt[3]{x} - 12x\sqrt[3]{x}$

$= (6x - 72x + 2x - 12x)\cdot\sqrt[3]{x}$

$= \boxed{-76x\sqrt[3]{x}}$

(2) $\dfrac{\sqrt{24}}{\sqrt{3}} = \sqrt{\dfrac{24}{3}} = \sqrt{8} = \sqrt{2\cdot 2\cdot 2} = \boxed{2\sqrt{2}}$

b) $\dfrac{3\sqrt{20}}{2\sqrt{10}} = \dfrac{3}{2}\cdot\sqrt{\dfrac{20}{10}} = \boxed{\dfrac{3\sqrt{2}}{2}}$

e) $\dfrac{1}{\sqrt{3}} + \dfrac{2}{\sqrt{6}}$ In this case we can rationalize BEFORE or AFTER

BEFORE:

$\dfrac{1}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

$\dfrac{2}{\sqrt{6}}\cdot\dfrac{\sqrt{6}}{\sqrt{6}} = \dfrac{2\sqrt{6}}{6}$

$\dfrac{\sqrt{3}}{3} + \dfrac{2\sqrt{6}}{6}$

LCD = 6

$\dfrac{2\sqrt{3}}{6} + \dfrac{2\sqrt{6}}{6} = \boxed{\dfrac{2\sqrt{3}+2\sqrt{6}}{6}}$

AFTER:

$\dfrac{1}{\sqrt{3}} + \dfrac{2}{\sqrt{6}} \rightarrow \dfrac{1\cdot\sqrt{2}}{\sqrt{6}} + \dfrac{2}{\sqrt{6}} = \dfrac{2+\sqrt{2}}{\sqrt{6}}$

LCD = $\sqrt{6}$

NOW MUST RATIONALIZE $\dfrac{2+\sqrt{2}}{\sqrt{6}}\cdot\dfrac{\sqrt{6}}{\sqrt{6}} = \dfrac{2\sqrt{6}+\sqrt{12}}{6} = \boxed{\dfrac{2\sqrt{6}+2\sqrt{3}}{6}}$

$\boxed{\dfrac{\sqrt{6}+\sqrt{3}}{3}}$

h) $\dfrac{3\sqrt{5}}{\sqrt{20}} + \dfrac{4\sqrt{3}}{\sqrt{27}}$ Will rationalize at the start

$$\frac{3\sqrt{5}}{\sqrt{20}} \cdot \frac{\sqrt{20}}{\sqrt{20}} = \frac{3\sqrt{100}}{20} = \frac{3\cdot(10)}{20} = \frac{30}{20}$$

$$\frac{4\sqrt{3}}{\sqrt{27}} \cdot \frac{\sqrt{27}}{\sqrt{27}} = \frac{4\sqrt{81}}{27} = \frac{4(9)}{27} = \frac{36}{27}$$

$$\frac{30}{20} + \frac{36}{27} = \frac{3}{2} + \frac{4}{3} = \frac{3\cdot3}{6} + \frac{4\cdot2}{6}$$

$$= \frac{9}{6} + \frac{8}{6}$$

$$= \boxed{\frac{17}{6}}$$

k) $\dfrac{2}{2\sqrt{3}+5}$ Must use DIFFERENCE OF SQUARES to rationalize.

$$\frac{2}{2\sqrt{3}+5}\cdot\frac{2\sqrt{3}-5}{2\sqrt{3}-5}=\frac{4\sqrt{3}-10}{2(2)\cdot\sqrt{3\cdot 3}-\cancel{10\sqrt{3}}+\cancel{10\sqrt{3}}-25}$$

$$=\frac{4\sqrt{3}-10}{4(3)-25}$$

$$=\frac{4\sqrt{3}-10}{-13}$$

$$=\boxed{\frac{10-4\sqrt{3}}{13}}$$

p) $$\frac{x^4+x^2}{\sqrt{x^3}}\cdot\frac{\sqrt{x^3}}{\sqrt{x^3}}=\frac{x^4\sqrt{x^3}+x^2\sqrt{x^3}}{x^3}$$

$$=\frac{x^4\sqrt{(x\cdot x)x}+x^2\sqrt{(x\cdot x)x}}{x^3}$$

$$=\frac{x^5\sqrt{x}+x^3\sqrt{x}}{x^3}=\frac{\cancel{x^5}^{2}\sqrt{x}}{\cancel{x^3}}+\frac{\cancel{x^3}\sqrt{x}}{\cancel{x^3}}$$

$$=x^2\sqrt{x}+\sqrt{x}=\boxed{\sqrt{x}\left(x^2+1\right)}$$

(3) $\sqrt{-3} \cdot \sqrt{-27} = 9$

TRUE

$\sqrt{(-3)\cdot(-27)} = \sqrt{+81} = 9$

(6) $V = L \cdot W \cdot H$

$\underline{\underline{L \cdot W}} = (4\sqrt{5} + 2\sqrt{3})(4\sqrt{5} - 2\sqrt{3})$

$= 16(5) - \cancel{8\sqrt{15}} + \cancel{8\sqrt{15}} - 4(3)$

$= 80 - 12$

$= 68$

$LW \cdot H = 68(3\sqrt{2} + 4)$

$= 68(3)\sqrt{2} + 68(4)$

$\boxed{V = 204\sqrt{2} + 272}$

1a) $\sqrt{3x+7} = 21$ Square both sides to rid of radical

$(\sqrt{3x+7})^2 = 21^2$

$3x + 7 = 441$

$3x = 441 - 7$

$\frac{3x}{3} = \frac{434}{3}$

$\boxed{x = \frac{434}{3}}$

c) $5 - \sqrt{2x-11} = 3$ Rearrange to Isolate Radical

$2 = \sqrt{2x-11}$ Square both sides

$2^2 = 2x - 11$

$\frac{4+11}{2} = \frac{2x}{2}$

$\boxed{x = \frac{15}{2}}$

e) $\sqrt{x} + 2 = x$ Rearrange to Isolate Radical

$\sqrt{x} = x - 2$ Square both sides

$x = (x-2)^2$

$x = x^2 - 4x + 4$

$x^2 - 5x + 4 = 0$ Product/Sum

$(x-4)(x-1) = 0$

$\boxed{x = 1, 4}$

g) $4 - x = \sqrt{x^2 - 8}$ Radical is already Isolated

$(4-x)^2 = x^2 - 8$ Square both sides

$(4-x)(4-x) = x^2 - 8$

$16 \overset{+8}{-} 8x + x^2 = x^2 \cancel{-8} \cancel{+8}$

$24 - 8x \cancel{+x^2} \cancel{-x^2} = \cancel{x^2} \cancel{-x^2}$

$24 - 8x = 0$

$\frac{8x}{8} = \frac{24}{8}$

$\boxed{x = 3}$

i) $\sqrt{1+9x} + 6 = 2x$ Rearrange to Isolate Radical

$\sqrt{1+9x} = 2x - 6$ Square both sides

$1 + 9x = (2x-6)^2$

$1 + 9x = (2x-6)(2x-6)$

$1^{-1} + 9x = 4x^2 - 24x + 36^{\,-1}$

$\overset{-9x}{\cancel{9x}} = 4x^2 \overset{-9x}{-} 24x + 35$

$4x^2 - 33x + 35 = 0$ Use Decomposition

140

Add	Mult
-33	+140
-72	-70 · -2
-39	-35 · -4
-33	-28 · -5 ✓

$4x^2 - 28x - 5x + 35$

$4x(x-7) - 5(x-7)$

$(4x-5)(x-7) = 0$

$\boxed{x = \frac{5}{4}, 7}$