2i) 41, 34, 27, 20...

-7 -7

$\boxed{T_1 = 41}$ (first term is T_1)

$\boxed{d = -7}$ (the difference between each consecutive term)

$T_7 = ?$ $\quad$ $T_n = T_1 + (n-1)d$

$T_7 = T_{n=7}$

$T_7 = 41 + (7-1)(-7)$

$T_7 = 41 + (-42)$

$\boxed{T_7 = -1}$

$T_{11} = T_{n=11}$

$T_{11} = 41 + (11-1)(-7)$

$T_{11} = 41 + (10)(-7)$

$\boxed{T_{11} = -29}$

2iv) $\frac{24}{5}, \frac{14}{5}, \frac{4}{5}, \frac{-6}{5}, \ldots$

getting smaller

$$T_1 = \frac{24}{5}$$

$$d = \frac{14}{5} - \frac{24}{5} = -\frac{10}{5}$$

$$d = -\frac{10}{5} \text{ or } -2$$

$T_7 = T_{n=7}$

$$T_n = T_1 + (n-1)d$$

$$T_7 = \frac{24}{5} + (7-1)\left(-\frac{10}{5}\right)$$

$$T_7 = \frac{24}{5} + (6)\left(-\frac{10}{5}\right) = \frac{24}{5} - \frac{60}{5}$$

$$T_7 = -\frac{36}{5}$$

5a) 5, 9, 13, ... 209

It is asking us for the number of term 'n' in this series.

$T_1 = 5$

$d = +4$

$T_n = 209$ (last term)

$n = ?$

$$T_n = T_1 + (n-1)d$$

I substitute the terms I know into the equation

$$209 = 5 + (n-1)(4)$$

1 equation, 1 unknown } I can solve for n!

$$209 = 5 + (n-1)(4)$$ (-5, -5)

$$\frac{204}{4} = \frac{(n-1)(4)}{4}$$

$$51 = n - 1$$

$$51 + 1 = n - 1 + 1$$

$$\boxed{n = 52}$$

2d) $\frac{4}{5}, \frac{2}{5}, -\frac{8}{15}, \ldots -\frac{126}{5}$

$T_1 = \frac{4}{5}$

$d = \frac{2}{5} - \frac{4}{5} = -\frac{2}{5}$

$d = -\frac{2}{5}$

$T_n = -\frac{126}{5}$ (the value of my n^{th} term)

$$T_n = T_1 + (n-1)d$$

I substitute what I know

$$-\frac{126}{5} = \frac{4}{5} + (n-1)\left(\frac{-2}{5}\right)$$

$$-\frac{126}{5} \overset{-\frac{4}{5}}{=} \frac{4}{5} \overset{-\frac{4}{5}}{} + (n-1)\left(\frac{-2}{5}\right)$$

$$-\frac{130}{5} = (n-1)\left(\frac{-2}{5}\right)$$

$$-26 \overset{\times -\frac{5}{2}}{=} (n-1)\left(\frac{-2}{5}\right) \times \frac{-5}{2}$$

$$65 = n - 1$$

$$\boxed{n = 66}$$

9) What is common difference 'd'?

Second Term is 4

$T_{n=2} = T_2$

$T_2 = 4$

$n = 2$

Eigth Term is 100

$T_{n=8} = T_8$

$T_8 = 100$

$n = 8$

$$T_n = T_1 + (n-1)d$$

Substitute what I know

$T_2 = T_1 + (2-1)d$

$4 = T_1 + (2-1)d$

$4 = T_1 + d$ (I have 2 unknowns. Need a second equation!)

$T_8 = T_1 + (8-1)d$

$100 = T_1 + (8-1)d$

$100 = T_1 + 7d$

Now I have 2 equations with 2 unknowns. I can solve! (Using substitution)

My 2 equations:

$4 = T_1 + d$ (this one looks easier to start)

$100 = T_1 + 7d$

$-T_1 \quad -T_1$

$4 = T_1 + d$

$d = 4 - T_1$

OR

$-d \quad -d$

$4 = T_1 + d$

$\boxed{T_1 = 4 - d}$

I think this will be easier to sub into other equation (no multiplying)

$100 = T_1 + 7d$

sub in $4-d$ for T_1

$100 = 4 - d + 7d$

$100 = 4 + 6d$

$100 - 4 = 4 - 4 + 6d$

$\frac{96}{6} = \frac{6d}{6}$

$\boxed{d = 16}$

I can now use my d value to solve T_1

$T_1 = 4 - d$

$T_1 = 4 - 16$

$\boxed{T_1 = -12}$