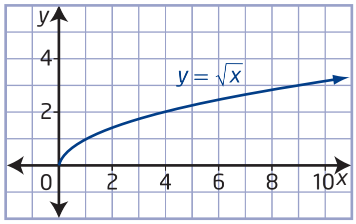
Key Ideas

- The base radical function is $y = \sqrt{x}$. Its graph has the following characteristics:
  - a left endpoint at (0, 0)
  - no right endpoint
  - the shape of half of a parabola
  - a domain of $\{x \mid x \geq 0, x \in \mathrm{R}\}$ and a range of $\{y \mid y \geq 0, y \in \mathrm{R}\}$



- You can graph radical functions of the form $y = a\sqrt{b(x - h)} + k$ by transforming the base function $y = \sqrt{x}$.
- You can analyse transformations to identify the domain and range of a radical function of the form $y = a\sqrt{b(x - h)} + k$.

How does each parameter affect the graph of $y = \sqrt{x}$?

Check Your Understanding

Practise

1. Graph each function using a table of values. Then, identify the domain and range.

a) $y = \sqrt{x - 1}$

b) $y = \sqrt{x + 6}$

c) $y = \sqrt{3 - x}$

d) $y = \sqrt{-2x - 5}$

2. Explain how to transform the graph of $y = \sqrt{x}$ to obtain the graph of each function. State the domain and range in each case.

a) $y = 7\sqrt{x - 9}$

b) $y = \sqrt{-x} + 8$

c) $y = -\sqrt{0.2x}$

d) $4 + y = \frac{1}{3}\sqrt{x + 6}$

3. Match each function with its graph.

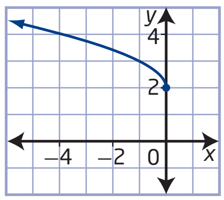
a) $y = \sqrt{x} - 2$

b) $y = \sqrt{-x} + 2$

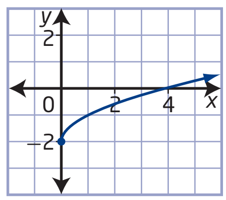
c) $y = -\sqrt{x + 2}$

d) $y = -\sqrt{-(x - 2)}$

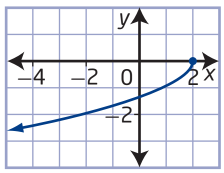
A



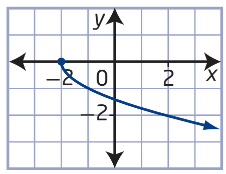
B



C



D



4. Write the equation of the radical function that results by applying each set of transformations to the graph of $y = \sqrt{x}$.

   a) vertical stretch by a factor of 4, then horizontal translation of 6 units left

   b) horizontal stretch by a factor of $\frac{1}{8}$, then vertical translation of 5 units down

   c) horizontal reflection in the y-axis, then horizontal translation of 4 units right and vertical translation of 11 units up

   d) vertical stretch by a factor of 0.25, vertical reflection in the x-axis, and horizontal stretch by a factor of 10

5. Sketch the graph of each function using transformations. State the domain and range of each function.

   a) $f(x) = \sqrt{-x} - 3$

   b) $r(x) = 3\sqrt{x + 1}$

   c) $p(x) = -\sqrt{x - 2}$

   d) $y - 1 = -\sqrt{-4(x - 2)}$

   e) $m(x) = \sqrt{\frac{1}{2}x + 4}$

   f) $y + 1 = \frac{1}{3}\sqrt{-(x + 2)}$

Apply

6. Consider the function $f(x) = \frac{1}{4}\sqrt{5x}$.

   a) Identify the transformations represented by $f(x)$ as compared to $y = \sqrt{x}$.

   b) Write two functions equivalent to $f(x)$: one of the form $y = a\sqrt{x}$ and the other of the form $y = \sqrt{bx}$

   c) Identify the transformation(s) represented by each function you wrote in part b).

   d) Use transformations to graph all three functions. How do the graphs compare?

7. **a)** Express the radius of a circle as a function of its area.

   b) Create a table of values and a graph to illustrate the relationship that this radical function represents.

8. For an observer at a height of h feet above the surface of Earth, the approximate distance, d, in miles, to the horizon can be modelled using the radical function $d = \sqrt{1.50h}$.

   a) Use the language of transformations to describe how to obtain the graph from the base square root graph.

   b) Determine an approximate equivalent function of the form $d = a\sqrt{h}$ for the function. Which form of the function do you prefer, and why?

   c) A lifeguard on a tower is looking out over the water with binoculars. How far can she see if her eyes are 20 ft above the level of the water? Express your answer to the nearest tenth of a mile.

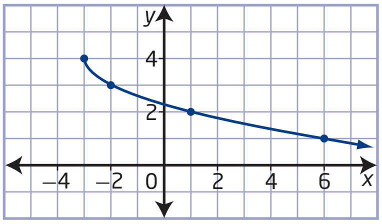
9. The function $4 - y = \sqrt{3x}$ is translated 9 units up and reflected in the x-axis.

   a) Without graphing, determine the domain and range of the image function.

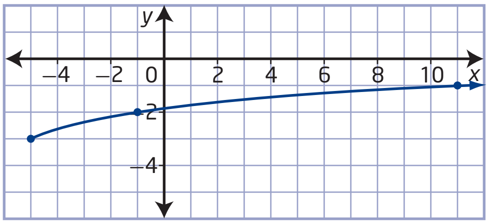
   b) Compared to the base function, $y = \sqrt{x}$, by how many units and in which direction has the given function been translated horizontally? vertically?

10. For each graph, write the equation of a radical function of the form $y = a\sqrt{b(x - h)} + k$.

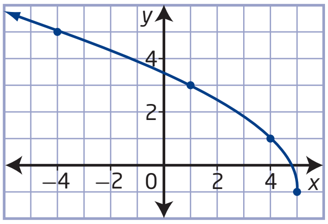
a)



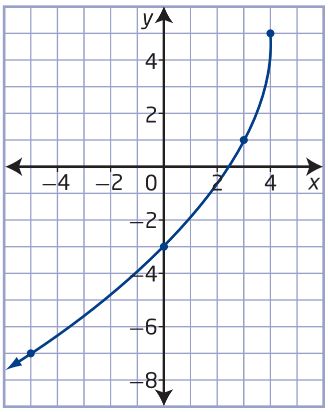
b)



c)



d)



11. Write the equation of a radical function with each domain and range.

a) $\{x \mid x \geq 6, x \in R\}$, $\{y \mid y \geq 1, y \in R\}$

b) $\{x \mid x \geq -7, x \in R\}$, $\{y \mid y \leq -9, y \in R\}$

c) $\{x \mid x \leq 4, x \in R\}$, $\{y \mid y \geq -3, y \in R\}$

d) $\{x \mid x \leq -5, x \in R\}$, $\{y \mid y \leq 8, y \in R\}$

12. Agronomists use radical functions to model and optimize corn production. One factor they analyse is how the amount of nitrogen fertilizer applied affects the crop yield. Suppose the function $Y(n) = 760\sqrt{n} + 2000$ is used to predict the yield, Y, in kilograms per hectare, of corn as a function of the amount, n, in kilograms per hectare, of nitrogen applied to the crop.

a) Use the language of transformations to compare the graph of this function to the graph of $y = \sqrt{n}$.

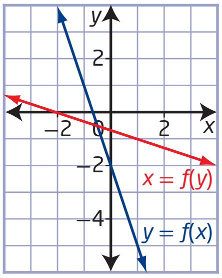
b) Graph the function using transformations.

c) Identify the domain and range.

d) What do the shape of the graph, the domain, and the range tell you about this situation? Are the domain and range realistic in this context? Explain.

Did You Know?

Over 6300 years ago, the Indigenous people in the area of what is now Mexico domesticated and cultivated several varieties of corn. The cultivation of corn is now global.

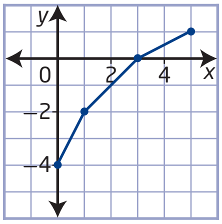
13. a) 

b) $y = x$, $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

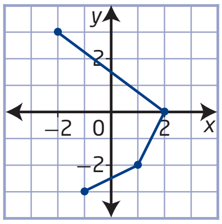
c) $f(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$
$f(y)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$

14.

$y = f(x)$		$y = f^{-1}(x)$	
x	y	x	y
−3	7	7	−3
2	4	4	2
10	−12	−12	10

15. a) 

The relation and its inverse are functions.

b) 

The relation is a function. The inverse is not a function.

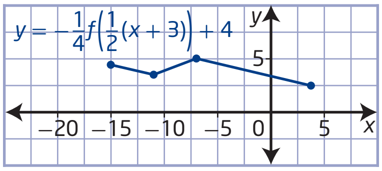
16. $y = \sqrt{x - 1} + 3$, restricted domain $\{x \mid x \geq 3, x \in R\}$

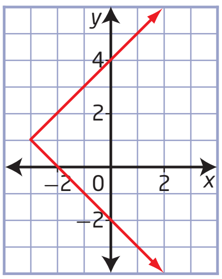
17. a) not inverses **b)** inverses

Chapter 1 Practice Test, pages 58 to 59

1. D **2.** D **3.** B **4.** B **5.** B **6.** C **7.** C

8. domain $\{x \mid -5 \leq x \leq 2, x \in R\}$

9. 

10. a) 

b) To transform it point by point, switch the position of the x- and the y-coordinate.

c) $(-1, -1)$

11. $y = \frac{1}{5}(x - 2)$

12. $y = 3f\left(-\frac{1}{2}(x - 2)\right)$

13. a) It is a translation of 2 units left and 7 units down.

b) $g(x) = |x + 2| - 7$ **c)** $(-2, -7)$

d) No. Invariant points are points that remain unchanged after a transformation.

14. a) $f(x) = x^2$

b) $g(x) = \frac{1}{4}f(x)$; a vertical stretch by a factor of $\frac{1}{4}$

c) $g(x) = f\left(\frac{1}{2}x\right)$; a horizontal stretch by a factor of 2

d) $\frac{1}{4}f(x) = \frac{1}{4}x^2$; $f\left(\frac{1}{2}x\right) = \left(\frac{1}{2}x\right)^2 = \frac{1}{4}x^2$

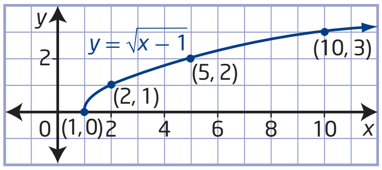
15. a) Using the horizontal line test, if a horizontal line passes through the function more than once the inverse is not a function.

b) $y = \pm\sqrt{-x - 5} - 3$

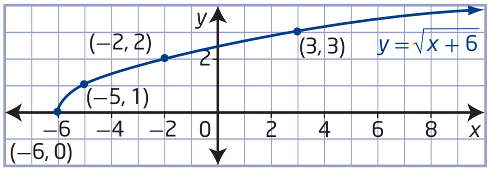
c) Example: restricted domain $\{x \mid x \geq -3, x \in R\}$

Chapter 2 Radical Functions

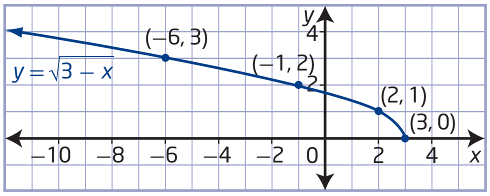
2.1 Radical Functions and Transformations, pages 72 to 77

1. a) 

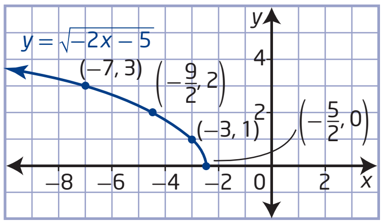
domain $\{x \mid x \geq 1, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$

b) 

domain $\{x \mid x \geq -6, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$

c) 

domain $\{x \mid x \leq 3, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$

d) 

domain $\left\{x \mid x \leq -\frac{5}{2}, x \in R\right\}$, range $\{y \mid y \geq 0, y \in R\}$

2. a) $a = 7 \rightarrow$ vertical stretch by a factor of 7
$h = 9 \rightarrow$ horizontal translation 9 units right
domain $\{x \mid x \geq 9, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$

b) $b = -1 \rightarrow$ reflected in y-axis
$k = 8 \rightarrow$ vertical translation up 8 units
domain $\{x \mid x \leq 0, x \in R\}$, range $\{y \mid y \geq 8, y \in R\}$

c) $a = -1 \rightarrow$ reflected in x-axis
$b = \frac{1}{5} \rightarrow$ horizontal stretch factor of 5
domain $\{x \mid x \geq 0, x \in R\}$, range $\{y \mid y \leq 0, y \in R\}$

d) $a = \frac{1}{3} \rightarrow$ vertical stretch factor of $\frac{1}{3}$
$h = -6 \rightarrow$ horizontal translation 6 units left
$k = -4 \rightarrow$ vertical translation 4 units down
domain $\{x \mid x \geq -6, x \in R\}$, range $\{y \mid y \geq -4, y \in R\}$

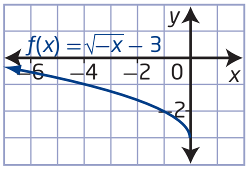
3. a) B **b)** A **c)** D **d)** C

4. a) $y = 4\sqrt{x + 6}$ **b)** $y = \sqrt{8x} - 5$

c) $y = \sqrt{-(x - 4)} + 11$ or $y = \sqrt{-x + 4} + 11$

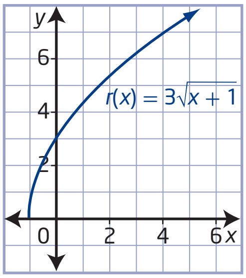
d) $y = -0.25\sqrt{0.1x}$ or $y = -\frac{1}{4}\sqrt{\frac{1}{10}x}$

5. a)



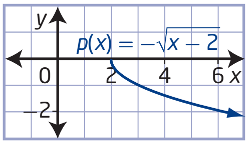
domain $\{x \mid x \leq 0, x \in R\}$, range $\{y \mid y \geq -3, y \in R\}$

b)



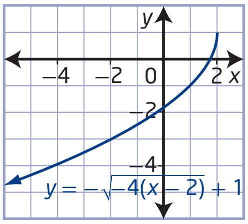
domain $\{x \mid x \geq -1, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$

c)



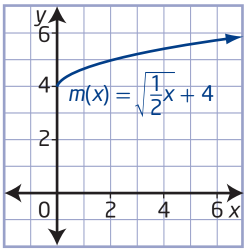
domain $\{x \mid x \geq 2, x \in R\}$, range $\{y \mid y \leq 0, y \in R\}$

d)



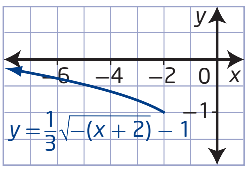
domain $\{x \mid x \leq 2, x \in R\}$, range $\{y \mid y \leq 1, y \in R\}$

e)



domain $\{x \mid x \geq 0, x \in R\}$, range $\{y \mid y \geq 4, y \in R\}$

f)



domain $\{x \mid x \leq -2, x \in R\}$, range $\{y \mid y \geq -1, y \in R\}$

6. a) $a = \frac{1}{4} \rightarrow$ vertical stretch factor of $\frac{1}{4}$

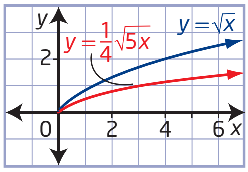
$b = 5 \rightarrow$ horizontal stretch factor of $\frac{1}{5}$

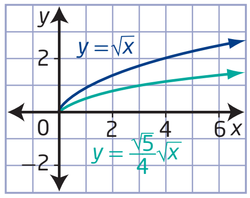
b) $y = \frac{\sqrt{5}}{4}\sqrt{x}$, $y = \sqrt{\frac{5}{16}x}$

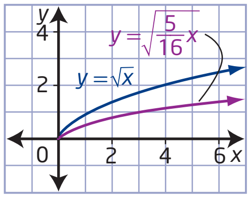
c) $a = \frac{\sqrt{5}}{4} \rightarrow$ vertical stretch factor of $\frac{\sqrt{5}}{4}$

$b = \frac{5}{16} \rightarrow$ horizontal stretch factor of $\frac{16}{5}$

d)





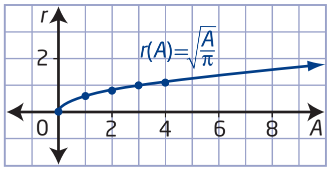


All graphs are the same.

7. a) $r(A) = \sqrt{\frac{A}{\pi}}$

b)

A	r
0	0
1	0.6
2	0.8
3	1.0
4	1.1



8. a) $b = 1.50 \rightarrow$ horizontal stretch factor of $\frac{1}{1.50}$ or $\frac{2}{3}$

b) $d \approx 1.22\sqrt{h}$ Example: I prefer the original function because the values are exact.

c) approximately 5.5 miles

9. a) domain $\{x \mid x \geq 0, x \in R\}$, range $\{y \mid y \geq -13, y \in R\}$

b) $h = 0 \rightarrow$ no horizontal translation

$k = 13 \rightarrow$ vertical translation down 13 units

10. a) $y = -\sqrt{x + 3} + 4$ **b)** $y = \frac{1}{2}\sqrt{x + 5} - 3$

c) $y = 2\sqrt{-(x - 5)} - 1$ or $y = 2\sqrt{-x + 5} - 1$

d) $y = -4\sqrt{-(x - 4)} + 5$ or $y = -4\sqrt{-x + 4} + 5$

11. Examples:

a) $y - 1 = \sqrt{x - 6}$ or $y = \sqrt{x - 6} + 1$

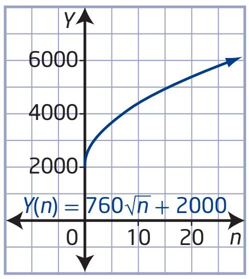
b) $y = -\sqrt{x + 7} - 9$ **c)** $y = 2\sqrt{-x + 4} - 3$

d) $y = -\sqrt{-(x + 5)} + 8$

12. a) $a = 760 \rightarrow$ vertical stretch factor of 760

$k = 2000 \rightarrow$ vertical translation up 2000

b)



c) domain $\{n \mid n \geq 0, n \in R\}$ range $\{Y \mid Y \geq 2000, Y \in R\}$

d) The minimum yield is 2000 kg/hectare. Example: The domain and range imply that the more nitrogen added, the greater the yield without end. This is not realistic.

13. a) domain $\{d \mid -100 \leq d \leq 0, d \in R\}$ range $\{P \mid 0 \leq P \leq 20, P \in R\}$ The domain is negative indicating days remaining, and the maximum value of P is 20 million.

b) $a = -2 \rightarrow$ reflected in d-axis, vertical stretch factor of 2; $b = -1 \rightarrow$ reflected in P-axis; $k = 20 \rightarrow$ vertical translation up 20 units.