

Key Ideas

- A sequence is an ordered list of elements.
- Elements within the range of the sequence are called terms of the sequence.
- To describe any term of a sequence, an expression is used for t_n , where $n \in \mathbb{N}$. This term is called the general term.
- In an arithmetic sequence, each successive term is formed by adding a constant. This constant is called the common difference.

- The general term of an arithmetic sequence is

$$t_n = t_1 + (n - 1)d$$

where t_1 is the first term

n is the number of terms ($n \in \mathbb{N}$)

d is the common difference

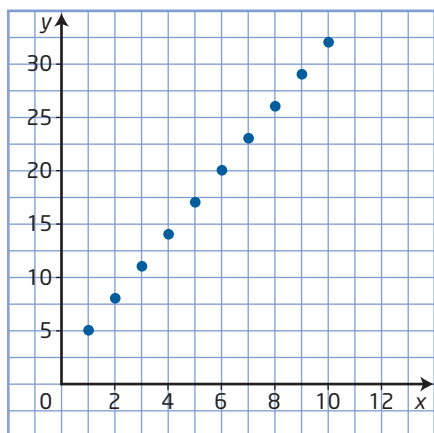
t_n is the general term or n th term

Check Your Understanding

Practise

1. Identify the arithmetic sequences from the following sequences. For each arithmetic sequence, state the value of t_1 , the value of d , and the next three terms.
 - a) 16, 32, 48, 64, 80, ...
 - b) 2, 4, 8, 16, 32, ...
 - c) -4, -7, -10, -13, -16, ...
 - d) 3, 0, -3, -6, -9, ...
2. Write the first four terms of each arithmetic sequence for the given values of t_1 and d .
 - a) $t_1 = 5$, $d = 3$
 - b) $t_1 = -1$, $d = -4$
 - c) $t_1 = 4$, $d = \frac{1}{5}$
 - d) $t_1 = 1.25$, $d = -0.25$
3. For the sequence defined by $t_n = 3n + 8$, find each indicated term.
 - a) t_1
 - b) t_7
 - c) t_{14}
4. For each arithmetic sequence determine the values of t_1 and d . State the missing terms of the sequence.
 - a) ■, ■, ■, 19, 23
 - b) ■, ■, 3, $\frac{3}{2}$
 - c) ■, 4, ■, ■, 10
5. Determine the position of the given term to complete the following statements.
 - a) 170 is the ■th term of -4, 2, 8, ...
 - b) -14 is the ■th term of $2\frac{1}{5}$, 2, $1\frac{4}{5}$, ...
 - c) 97 is the ■th term of -3, 1, 5, ...
 - d) -10 is the ■th term of 14, 12.5, 11, ...
6. Determine the second and third terms of an arithmetic sequence if
 - a) the first term is 6 and the fourth term is 33
 - b) the first term is 8 and the fourth term is 41
 - c) the first term is 42 and the fourth term is 27

7. The graph of an arithmetic sequence is shown.

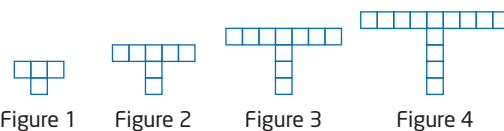


- What are the first five terms of the sequence?
- Write the general term of this sequence.
- What is t_{50} ? t_{200} ?
- Describe the relationship between the slope of the graph and your formula from part b).
- Describe the relationship between the y -intercept and your formula from part b).

Apply

- Which arithmetic sequence(s) contain the term 34? Justify your conclusions.
 - $t_n = 6 + (n - 1)4$
 - $t_n = 3n - 1$
 - $t_1 = 12, d = 5.5$
 - 3, 7, 11, ...
- Determine the first term of the arithmetic sequence in which the 16th term is 110 and the common difference is 7.
- The first term of an arithmetic sequence is $5y$ and the common difference is $-3y$. Write the equations for t_n and t_{15} .
- The terms $5x + 2$, $7x - 4$, and $10x + 6$ are consecutive terms of an arithmetic sequence. Determine the value of x and state the three terms.

- The numbers represented by x , y , and z are the first three terms of an arithmetic sequence. Express z in terms of x and y .
- Each square in this pattern has a side length of 1 unit. Assume the pattern continues.



- Write an equation in which the perimeter is a function of the figure number.
 - Determine the perimeter of Figure 9.
 - Which figure has a perimeter of 76 units?
- The Wolf Creek Golf Course, located near Ponoka, Alberta, has been the site of the Canadian Tour Alberta Open Golf Championship. This tournament has a maximum entry of 132 players. The tee-off times begin at 8:00 and are 8 min apart.
 - The tee-off times generate an arithmetic sequence. Write the first four terms of the arithmetic sequence, if the first tee-off time of 8:00 is considered to be at time 0.
 - Following this schedule, how many players will be on the course after 1 h, if the tee-off times are for groups of four?
 - Write the general term for the sequence of tee-off times.
 - At what time will the last group tee-off?
 - What factors might affect the prearranged tee-off time?

Did You Know?

The first championship at Wolf Creek was held in 1987 and has attracted PGA professionals, including Mike Weir and Dave Barr.

Answers

Chapter 1 Sequences and Series

1.1 Arithmetic Sequences, pages 16 to 21

1. a) arithmetic sequence: $t_1 = 16$, $d = 16$; next three terms: 96, 112, 128
 b) not arithmetic
 c) arithmetic sequence: $t_1 = -4$, $d = -3$; next three terms: -19, -22, -25
 d) arithmetic sequence: $t_1 = 3$, $d = -3$; next three terms: -12, -15, -18
2. a) 5, 8, 11, 14 b) -1, -5, -9, -13
 c) $4, \frac{21}{5}, \frac{22}{5}, \frac{23}{5}$ d) 1.25, 1.00, 0.75, 0.50
3. a) $t_1 = 11$ b) $t_7 = 29$ c) $t_{14} = 50$
4. a) 7, 11, 15, 19, 23; $t_1 = 7$, $d = 4$
 b) $6, \frac{9}{2}, 3, \frac{3}{2}$; $t_1 = 6$, $d = -\frac{3}{2}$
 c) 2, 4, 6, 8, 10; $t_1 = 2$, $d = 2$
5. a) 30 b) 82 c) 26 d) 17
6. a) $t_2 = 15$, $t_3 = 24$ b) $t_2 = 19$, $t_3 = 30$
 c) $t_2 = 37$, $t_3 = 32$
7. a) 5, 8, 11, 14, 17 b) $t_n = 3n + 2$
 c) $t_{50} = 152$, $t_{200} = 602$
 d) The general term is a linear equation of the form $y = mx + b$, where $t_n = y$ and $n = x$. Therefore, $t_n = 3n + 2$ has a slope of 3.
 e) The constant value of 2 in the general term is the y -intercept of 2.
8. A and C; both sequences have a natural-number value for n .
9. 5
10. $t_n = -3yn + 8y$; $t_{15} = -37y$
11. $x = -16$; first three terms: -78, -116, -154
12. $z = 2y - x$
13. a) $t_n = 6n + 4$ b) 58
 c) 12
14. a) 0, 8, 16, 24
 b) 32 players
 c) $t_n = 8n - 8$
 d) 12:16
 e) Example: weather, all foursomes starting on time, etc.
15. 21 square inches
16. a) $t_n = 2n - 1$ b) 51st day
 c) Susan continues the program until she accomplishes her goal.
17. a)

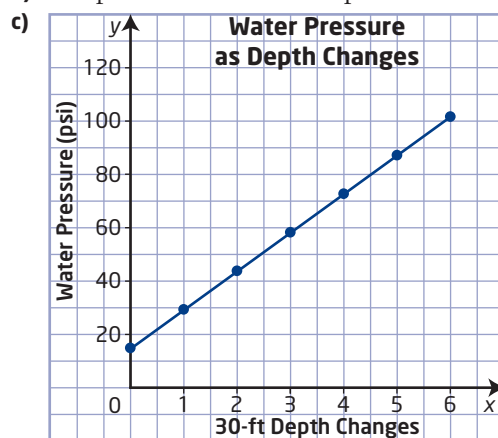
Carbon Atoms	1	2	3	4
Hydrogen Atoms	4	6	8	10

 b) $t_n = 2n + 2$ or $H = 2C + 2$
 c) 100 carbon atoms

18.

Multiples of	28	7	15
Between	1 and 1000	500 and 600	50 and 500
First Term, t_1	28	504	60
Common Difference, d	28	7	15
n th Term, t_n	980	595	495
General Term	$t_n = 28n$	$t_n = 7n + 497$	$t_n = 15n + 45$
Number of Terms	35	14	30

19. a) 14.7, 29.4, 44.1, 58.8; $t_n = 14.7n$, where n represents every increment of 30 ft in depth.
 b) 490 psi at 1000 ft and 980 psi at 2000 ft



- d) 14.7 psi
- e) 14.7
- f) The y -intercept represents the first term of the sequence and the slope represents the common difference.
20. Other lengths are 6 cm, 12 cm, and 18 cm. Add the four terms to find the perimeter. Replace t_2 with $t_1 + d$, t_3 with $t_1 + 2d$, and t_4 with $t_1 + 3d$. Solve for d .
21. a) 4, 8, 12, 16, 20 b) $t_n = 4n$
 c) 320 min
22. -29 beekeepers
23. 5.8 million carats. This value represents the increase of diamond carats mined each year.
24. 1696.5 m
25. a) 13:54, 13:59, 14:04, 14:09, 14:14; $t_1 = 13:54$, $d = 0:05$
 b) $t_n = 0:05n + 13:49$
 c) Assume that the arithmetic sequence of times continues.
 d) 15:49