Key Ideas

- Given the sequence $t_1, t_2, t_3, t_4, \ldots, t_n$ the associated series is $S_n = t_1 + t_2 + t_3 + t_4 + \cdots + t_n$.
- For the general arithmetic series,
  $t_1 + (t_1 + d) + (t_1 + 2d) + \cdots + (t_1 + [n - 1]d)$ or
  $t_1 + (t_1 + d) + (t_1 + 2d) + \cdots + (t_n - d) + t_n$,
  the sum of the first n terms is
  $S_n = \frac{n}{2}[2t_1 + (n - 1)d]$ or $S_n = \frac{n}{2}(t_1 + t_n)$,
  where t_1 is the first term
  n is number of terms
  d is the common difference
  t_n is the nth term
  S_n is the sum to n terms

Check Your Understanding

Practise

1. Determine the sum of each arithmetic series.
   a) $5 + 8 + 11 + \cdots + 53$
   b) $7 + 14 + 21 + \cdots + 98$
   c) $8 + 3 + (-2) + \cdots + (-102)$
   d) $\frac{2}{3} + \frac{5}{3} + \frac{8}{3} + \cdots + \frac{41}{3}$
2. For each of the following arithmetic series, determine the values of t_1 and d, and the value of S_n to the indicated sum.
   a) $1 + 3 + 5 + \cdots (S_8)$
   b) $40 + 35 + 30 + \cdots (S_{11})$
   c) $\frac{1}{2} + \frac{3}{2} + \frac{5}{2} + \cdots (S_7)$
   d) $(-3.5) + (-1.25) + 1 + \cdots (S_6)$
3. Determine the sum, S_n, for each arithmetic sequence described.
   a) $t_1 = 7$, $t_n = 79$, $n = 8$
   b) $t_1 = 58$, $t_n = -7$, $n = 26$
   c) $t_1 = -12$, $t_n = 51$, $n = 10$
   d) $t_1 = 12$, $d = 8$, $n = 9$
   e) $t_1 = 42$, $d = -5$, $n = 14$
4. Determine the value of the first term, t_1, for each arithmetic series described.
   a) $d = 6$, $S_n = 574$, $n = 14$
   b) $d = -6$, $S_n = 32$, $n = 13$
   c) $d = 0.5$, $S_n = 218.5$, $n = 23$
   d) $d = -3$, $S_n = 279$, $n = 18$
5. For the arithmetic series, determine the value of n.
   a) $t_1 = 8$, $t_n = 68$, $S_n = 608$
   b) $t_1 = -6$, $t_n = 21$, $S_n = 75$
6. For each series find t_{10} and S_{10}.
   a) $5 + 10 + 15 + \cdots$
   b) $10 + 7 + 4 + \cdots$
   c) $(-10) + (-14) + (-18) + \cdots$
   d) $2.5 + 3 + 3.5 + \cdots$

Apply

7. a) Determine the sum of all the multiples of 4 between 1 and 999.
   b) What is the sum of the multiples of 6 between 6 and 999?

8. It's About Time, in Langley, British Columbia, is Canada's largest custom clock manufacturer. They have a grandfather clock that, on the hours, chimes the number of times that corresponds to the time of day. For example, at 4:00 p.m., it chimes 4 times. How many times does the clock chime in a 24-h period?

9. A training program requires a pilot to fly circuits of an airfield. Each day, the pilot flies three more circuits than the previous day. On the fifth day, the pilot flew 14 circuits. How many circuits did the pilot fly

a) on the first day?

b) in total by the end of the fifth day?

c) in total by the end of the nth day?

10. The second and fifth terms of an arithmetic series are 40 and 121, respectively. Determine the sum of the first 25 terms of the series.

11. The sum of the first five terms of an arithmetic series is 85. The sum of the first six terms is 123. What are the first four terms of the series?

12. Galileo noticed a relationship between the distance travelled by a falling object and time. Suppose data show that when an object is dropped from a particular height it moves approximately 5 m during the first second of its fall, 15 m during the second second, 25 m during the third second, 35 during the fourth second, and so on. The formula describing the approximate distance, d, the object is from its starting position n seconds after it has been dropped is $d(n) = 5n^2$.

a) Using the general formula for the sum of a series, derive the formula $d(n) = 5n^2$.

b) Demonstrate algebraically, using $n = 100$, that the sum of the series $5 + 15 + 25 + \cdots$ is equivalent to $d(n) = 5n^2$.

13. At the sixth annual Vancouver Canstruction® Competition, architects and engineers competed to see whose team could build the most spectacular structure using little more than cans of food.

A Breach in Hunger

The UnBEARable Truth

Stores often stack cans for display purposes, although their designs are not usually as elaborate as the ones shown above. To calculate the number of cans in a display, an arithmetic series may be used. Suppose a store wishes to stack the cans in a pattern similar to the one shown. This display has one can at the top and each row thereafter adds one can. If there are 18 rows, how many cans in total are there in the display?

Did You Know?

The Vancouver Canstruction® Competition aids in the fight against hunger. At the end of the competition, all canned food is donated to food banks.

26. a) $d > 0$ **b)** $d < 0$
c) $d = 0$ **d)** t_1
e) t_n

27. Definition: An ordered list of terms in which the difference between consecutive terms is constant.
Common Difference: The difference between successive terms, $d = t_n - t_{n-1}$
Example: 12, 19, 26, …
Formula: $t_n = 7n + 5$

28. Step 1 The graph of an arithmetic sequence is always a straight line. The common difference is described by the slope of the graph. Since the common difference is always constant, the graph will be a straight line.
Step 2
a) Changing the value of the first term changes the y-intercept of the graph. The y-intercept increases as the value of the first term increases. The y-intercept decreases as the value of the first term decreases.
b) Yes, the graph keeps it shape. The slope stays the same.

Step 3
a) Changing the value of the common difference changes the slope of the graph.
b) As the common difference increases, the slope increases. As the common difference decreases, the slope decreases.

Step 4 The common difference is the slope.
Step 5 The slope of the graph represents the common difference of the general term of the sequence. The slope is the coefficient of the variable n in the general term of the sequence.

1.2 Arithmetic Series, pages 27 to 31

1. a) 493 **b)** 735
c) -1081 **d)** $\frac{301}{3} = 100.\overline{3}$

2. a) $t_1 = 1$, $d = 2$, $S_8 = 64$
b) $t_1 = 40$, $d = -5$, $S_{11} = 165$
c) $t_1 = \frac{1}{2}$, $d = 1$, $S_7 = 24.5$
d) $t_1 = -3.5$, $d = 2.25$, $S_6 = 12.75$

3. a) 344 **b)** 663
c) 195 **d)** 396
e) 133

4. a) 2 **b)** $\frac{500}{13} \approx 38.46$
c) 4 **d)** 41

5. a) 16 **b)** 10

6. a) $t_{10} = 50$, $S_{10} = 275$
b) $t_{10} = -17$, $S_{10} = -35$
c) $t_{10} = -46$, $S_{10} = -280$
d) $t_{10} = 7$, $S_{10} = 47.5$

7. a) 124 500 **b)** 82 665

8. 156 times

9. a) 2 **b)** 40 **c)** $\frac{n}{2}(1 + 3n)$

10. 8425

11. $3 + 10 + 17 + 24$

12. a)
$$S_n = \frac{n}{2}[2t_1 + (n - 1)d]$$
$$S_n = \frac{n}{2}[2(5) + (n - 1)10]$$
$$S_n = \frac{n}{2}[10 + 10n - 10]$$
$$S_n = \frac{n(10n)}{2}$$
$$S_n = \frac{10n^2}{2}$$
$$S_n = 5n^2$$

b)
$$S_{100} = \frac{100}{2}[2(5) + (100 - 1)10]$$
$$S_{100} = \frac{100}{2}[10 + 990]$$
$$S_{100} = \frac{100}{2}(1000)$$
$$S_{100} = 50\,000$$
$$d(100) = 5(100)^2$$
$$d(100) = 5(10\,000)$$
$$d(100) = 50\,000$$

13. 171

14. a) the number of handshakes between six people if they each shake hands once
b) $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9$
c) 435
d) Example: The number of games played in a home and away series league for n teams.

15. a) $t_1 = 6.2$, $d = 1.2$
b) $t_{20} = 29$
c) $S_{20} = 352$

16. 173 cm

17. a) True. Example: $2 + 4 + 6 + 8 = 20$, $4 + 8 + 12 + 16 = 40$, $40 = 2 \times 20$
b) False. Example: $2 + 4 + 6 + 8 = 20$, $2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 = 72$, $72 \neq 2 \times 20$
c) True. Example: Given the sequence 2, 4, 6, 8, multiplying each term by 5 gives 10, 20, 30, 40. Both sequences are arithmetic sequences.

18. a) $7 + 11 + 15$ **b)** 250 **c)** 250
d)
$$S_n = \frac{n}{2}[2t_1 + (n - 1)d]$$
$$S_n = \frac{n}{2}[2(7) + (n - 1)4]$$
$$S_n = \frac{n}{2}[14 + 4n - 4]$$
$$S_n = \frac{n}{2}[4n + 10]$$
$$S_n = n(2n + 5)$$
$$S_n = 2n^2 + 5n$$