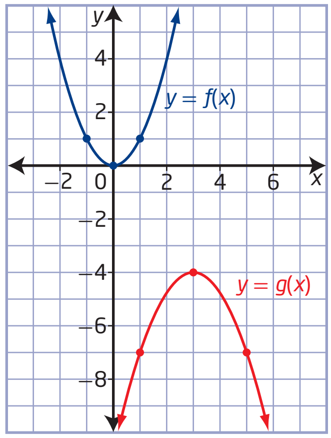
Your Turn

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. State the equation of the transformed function. Explain your answer.



Key Ideas

- Write the function in the form $y = af(b(x - h)) + k$ to better identify the transformations.
- Stretches and reflections may be performed in any order before translations.
- The parameters a, b, h, and k in the function $y = af(b(x - h)) + k$ correspond to the following transformations:
  - a corresponds to a vertical stretch about the x-axis by a factor of $|a|$. If $a < 0$, then the function is reflected in the x-axis.
  - b corresponds to a horizontal stretch about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the function is reflected in the y-axis.
  - h corresponds to a horizontal translation.
  - k corresponds to a vertical translation.

Check Your Understanding

Practise

1. The function $y = x^2$ has been transformed to $y = af(bx)$. Determine the equation of each transformed function.

a) Its graph is stretched horizontally about the y-axis by a factor of 2 and then reflected in the x-axis.

b) Its graph is stretched horizontally about the y-axis by a factor of $\frac{1}{4}$, reflected in the y-axis, and then stretched vertically about the x-axis by a factor of $\frac{1}{4}$.

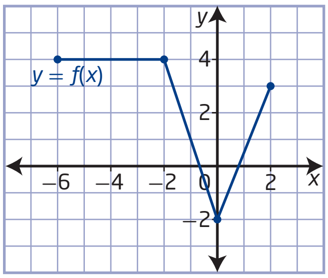
2. The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks.

The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the ■ by a factor of ■. It is vertically stretched about the ■ by a factor of ■. It is reflected in the ■, and then translated ■ units to the right and ■ units down.

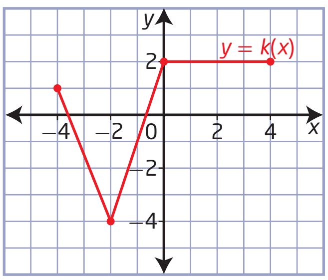
3. Copy and complete the table by describing the transformations of the given functions, compared to the function $y = f(x)$.

Function	Reflections	Vertical Stretch Factor	Horizontal Stretch Factor	Vertical Translation	Horizontal Translation
$y - 4 = f(x - 5)$					
$y + 5 = 2f(3x)$					
$y = \frac{1}{2}f\left(\frac{1}{2}(x - 4)\right)$					
$y + 2 = -3f(2(x + 2))$					

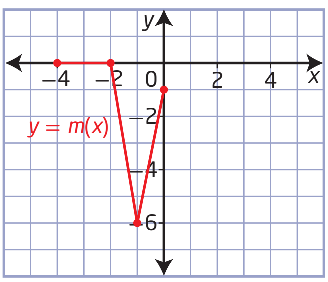
4. Using the graph of $y = f(x)$, write the equation of each transformed graph in the form $y = af(b(x - h)) + k$.



a)

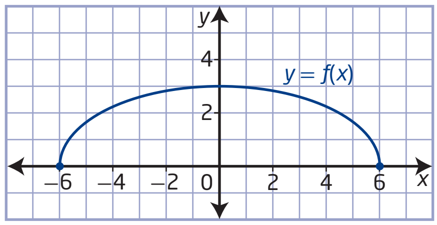


b)



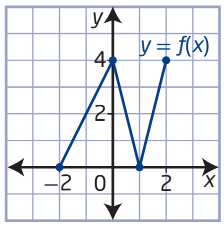
5. For each graph of $y = f(x)$, sketch the graph of the combined transformations. Show each transformation in the sequence.

a)



- vertical stretch about the x-axis by a factor of 2
- horizontal stretch about the y-axis by a factor of $\frac{1}{3}$
- translation of 5 units to the left and 3 units up

b)



- vertical stretch about the x-axis by a factor of $\frac{3}{4}$
- horizontal stretch about the y-axis by a factor of 3
- translation of 3 units to the right and 4 units down

6. The key point $(-12, 18)$ is on the graph of $y = f(x)$. What is its image point under each transformation of the graph of $f(x)$?

a) $y + 6 = f(x - 4)$

b) $y = 4f(3x)$

c) $y = -2f(x - 6) + 4$

d) $y = -2f\left(-\frac{2}{3}x - 6\right) + 4$

e) $y + 3 = -\frac{1}{3}f(2(x + 6))$

Apply

7. Describe, using an appropriate order, how to obtain the graph of each function from the graph of $y = f(x)$. Then, give the mapping for the transformation.

a) $y = 2f(x - 3) + 4$

b) $y = -f(3x) - 2$

c) $y = -\frac{1}{4}f(-(x + 2))$

d) $y - 3 = -f(4(x - 2))$

e) $y = -\frac{2}{3}f\left(-\frac{3}{4}x\right)$

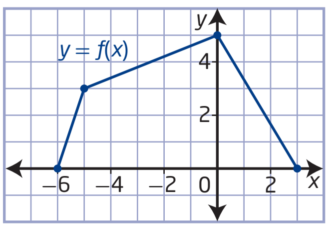
f) $3y - 6 = f(-2x + 12)$

8. Given the function $y = f(x)$, write the equation of the form $y - k = af(b(x - h))$ that would result from each combination of transformations.

a) a vertical stretch about the x-axis by a factor of 3, a reflection in the x-axis, a horizontal translation of 4 units to the left, and a vertical translation of 5 units down

b) a horizontal stretch about the y-axis by a factor of $\frac{1}{3}$, a vertical stretch about the x-axis by a factor of $\frac{3}{4}$, a reflection in both the x-axis and the y-axis, and a translation of 6 units to the right and 2 units up

9. The graph of $y = f(x)$ is given. Sketch the graph of each of the following functions.



a) $y + 2 = f(x - 3)$

b) $y = -f(-x)$

c) $y = f(3(x - 2)) + 1$

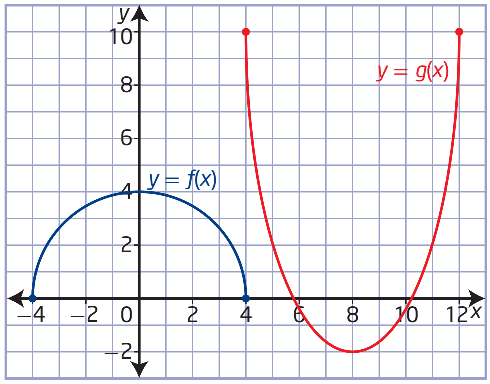
d) $y = 3f\left(\frac{1}{3}x\right)$

e) $y + 2 = -3f(x + 4)$

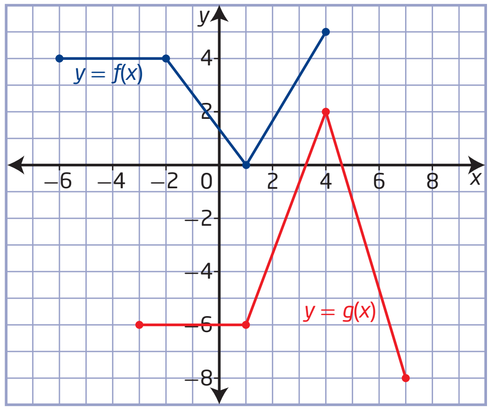
f) $y = \frac{1}{2}f\left(-\frac{1}{2}(x + 2)\right) - 1$

10. The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.

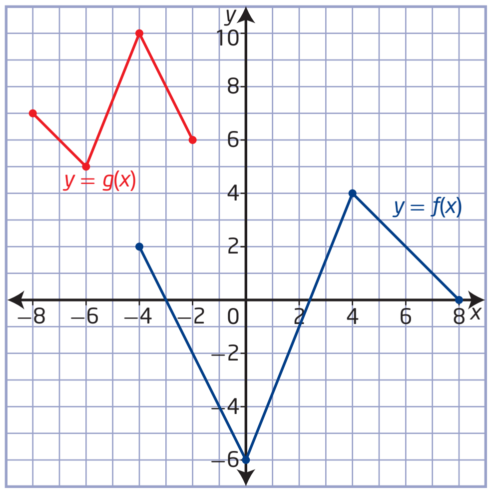
a)



b)



c)



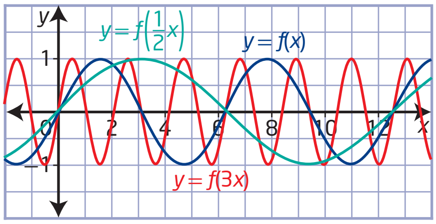
11. Given the function $f(x)$, sketch the graph of the transformed function $g(x)$.

a) $f(x) = x^2$, $g(x) = -2f(4(x + 2)) - 2$

b) $f(x) = |x|$, $g(x) = -2f(-3x + 6) + 4$

c) $f(x) = x$, $g(x) = -\frac{1}{3}f(-2(x + 3)) - 2$

C4



C5 a) $t_n = 4n - 14$ **b)** $t_n = -4n + 14$

c) They are reflections of each other in the x-axis.

1.3 Combining Transformations, pages 38 to 43

1. a) $y = -f\left(\frac{1}{2}x\right)$ or $y = -\frac{1}{4}x^2$

b) $y = \frac{1}{4}f(-4x)$ or $y = 4x^2$

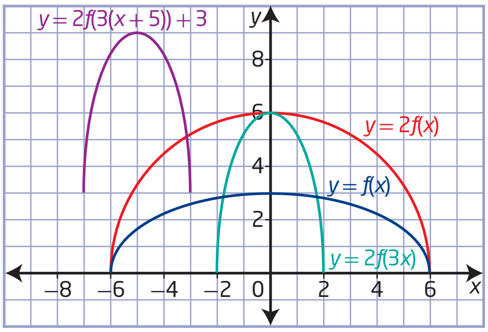
2. The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the y-axis by a factor of $\frac{1}{4}$. It is vertically stretched about the x-axis by a factor of 3. It is reflected in the x-axis, and then translated 4 units right and 10 units down.

3.

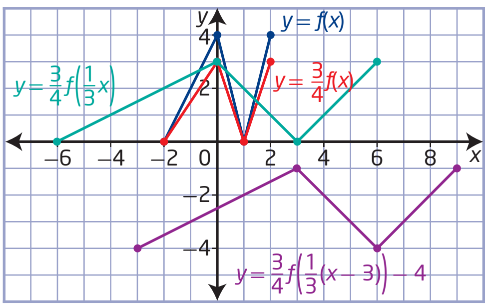
Function	Reflections	Vertical Stretch Factor	Horizontal Stretch Factor	Vertical Translation	Horizontal Translation
$y - 4 = f(x - 5)$	none	none	none	4	5
$y + 5 = 2f(3x)$	none	2	$\frac{1}{3}$	−5	none
$y = \frac{1}{2}f\left(\frac{1}{2}(x - 4)\right)$	none	$\frac{1}{2}$	2	none	4
$y + 2 = -3f(2(x + 2))$	x-axis	3	$\frac{1}{2}$	−2	−2

4. a) $y = f(-(x + 2)) - 2$ **b)** $y = f(2(x + 1)) - 4$

5. a)



b)



6. a) (−8, 12) **b)** (−4, 72) **c)** (−6, −32)

d) (9, −32) **e)** (−12, −9)

7. a) vertical stretch by a factor of 2 and translation of 3 units right and 4 units up; $(x, y) \rightarrow (x + 3, 2y + 4)$

b) horizontal stretch by a factor of $\frac{1}{3}$, reflection in the x-axis, and translation of 2 units down; $(x, y) \rightarrow \left(\frac{1}{3}x, -y - 2\right)$

c) reflection in the y-axis, reflection in the x-axis, vertical stretch by a factor of $\frac{1}{4}$, and translation of 2 units left; $(x, y) \rightarrow \left(-x - 2, -\frac{1}{4}y\right)$

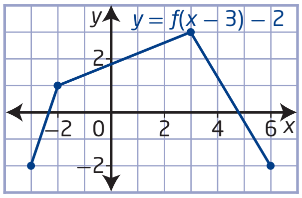
d) horizontal stretch by a factor of $\frac{1}{4}$, reflection in the x-axis, and translation of 2 units right and 3 units up; $(x, y) \rightarrow \left(\frac{1}{4}x + 2, -y + 3\right)$

e) reflection in the y-axis, horizontal stretch by a factor of $\frac{4}{3}$, reflection in the x-axis, and vertical stretch by a factor of $\frac{2}{3}$; $(x, y) \rightarrow \left(-\frac{4}{3}x, -\frac{2}{3}y\right)$

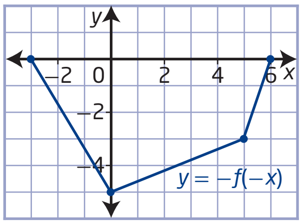
f) reflection in the y-axis, horizontal stretch by a factor of $\frac{1}{2}$, vertical stretch by a factor of $\frac{1}{3}$, and translation of 6 units right and 2 units up; $(x, y) \rightarrow \left(-\frac{1}{2}x + 6, \frac{1}{3}y + 2\right)$

8. a) $y + 5 = -3f(x + 4)$ **b)** $y - 2 = -\frac{3}{4}f(-3(x - 6))$

9. a)



b)



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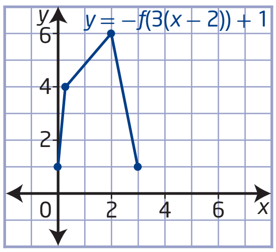
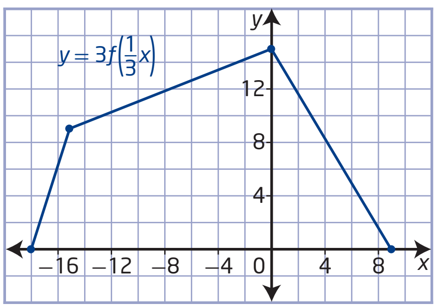
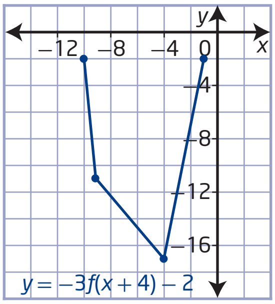
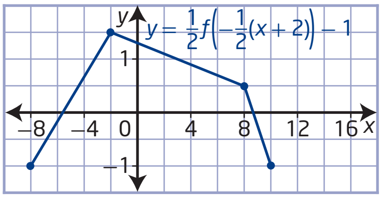


d)

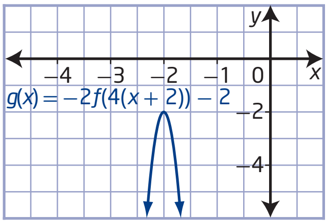


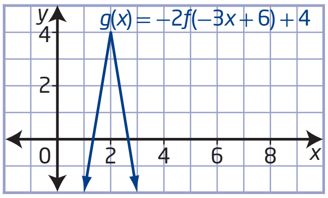
e) 

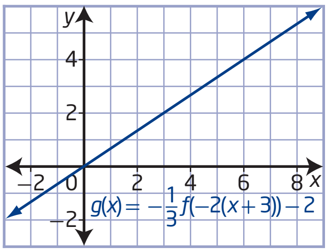
f) 

10. a) $y = -3f(x - 8) + 10$ **b)** $y = -2f(x - 3) + 2$

c) $y = -\frac{1}{2}f(-2(x + 4)) + 7$

11. a) 

b) 

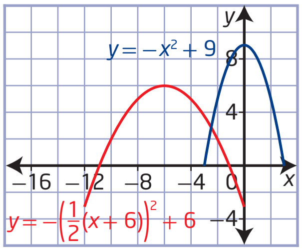
c) 

12. a) A′(−11, −2), B′(−7, 6), C′(−3, 4), D′(−1, 5), E′(3, −2)

b) $y = -f\left(\frac{1}{2}(x + 3)\right) + 4$

13. a) The graphs are in two locations because the transformations performed to obtain Graph 2 do not match those in $y = |2x - 6| + 2$. Gil forgot to factor out the coefficient of the x-term, 2, from −6. The horizontal translation should have been 3 units right, not 6 units.

b) He should have rewritten the function as $y = |2(x - 3)| + 2$.

14. a) 

b) $y = -\left(\frac{1}{2}(x + 6)\right)^2 + 6$

15. a) $(-a, 0)$, $(0, -b)$ **b)** $(2a, 0)$, $(0, 2b)$

c) and **d)** There is not enough information to determine the locations of the new intercepts. When a transformation involves translations, the locations of the new intercepts will vary with different base functions.

16. a) $A = -2x^3 + 18x$ **b)** $A = -\frac{1}{8}x^3 + 18x$

c) For (2, 5), the area of the rectangle in part a) is 20 square units.
$A = -2x^3 + 18x$
$A = -2(2)^3 + 18(2)$
$A = 20$

For (8, 5), the area of the rectangle in part b) is 80 square units.
$A = -\frac{1}{8}x^3 + 18x$
$A = -\frac{1}{8}(8)^3 + 18(8)$
$A = 80$

17. $y = 36(x - 2)^2 + 6(x - 2) - 2$

18. Example: vertical stretches and horizontal stretches followed by reflections

C1 **Step 1** They are reflections in the axes.
1: $y = x + 3$, 2: $y = -x - 3$, 3: $y = x - 3$
Step 2 They are vertical translations coupled with reflections. 1: $y = x^2 + 1$, 2: $y = x^2 - 1$, 3: $y = -x^2$, 4: $y = -x^2 - 1$

C2 a) The cost of making $b + 12$ bracelets, and it is a horizontal translation.

b) The cost of making b bracelets plus 12 more dollars, and it is a vertical translation.

c) Triple the cost of making b bracelets, and it is a vertical stretch.

d) The cost of making $\frac{b}{2}$ bracelets, and it is a horizontal stretch.

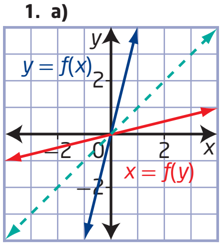
C3 $y = 2(x - 3)^2 + 1$; a vertical stretch by a factor of 2 and a translation of 3 units right and 1 unit up

C4 a) H is repeated; J is transposed; K is repeated and transposed

b) H is in retrograde; J is inverted; K is in retrograde and inverted

c) H is inverted, repeated, and transposed; J is in retrograde inversion and repeated; K is in retrograde and transposed

1.4 Inverse of a Relation, pages 51 to 55

1. a) 

b)