Key Ideas

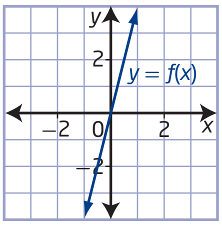
- You can find the inverse of a relation by interchanging the x-coordinates and y-coordinates of the graph.
- The graph of the inverse of a relation is the graph of the relation reflected in the line $y = x$.
- The domain and range of a relation become the range and domain, respectively, of the inverse of the relation.
- Use the horizontal line test to determine if an inverse will be a function.
- You can create an inverse that is a function over a specified interval by restricting the domain of a function.
- When the inverse of a function $f(x)$ is itself a function, it is denoted by $f^{-1}(x)$.
- You can verify graphically whether two functions are inverses of each other.

Check Your Understanding

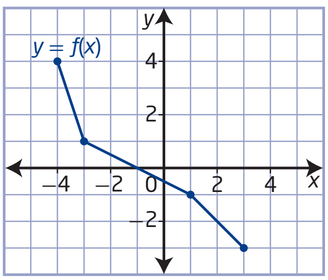
Practise

1. Copy each graph. Use the reflection line $y = x$ to sketch the graph of $x = f(y)$ on the same set of axes.

a)

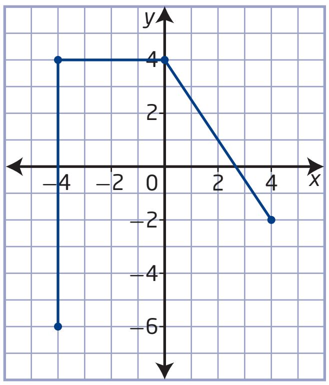


b)

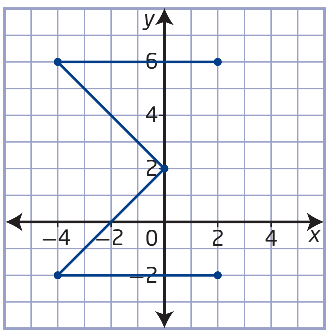


2. Copy the graph of each relation and sketch the graph of its inverse relation.

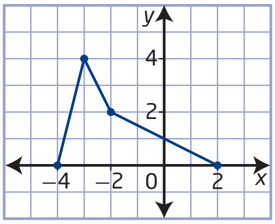
a)

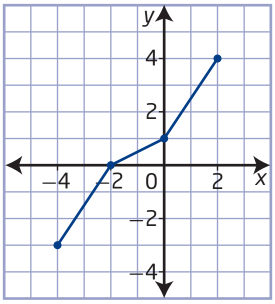


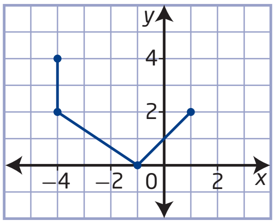
b)



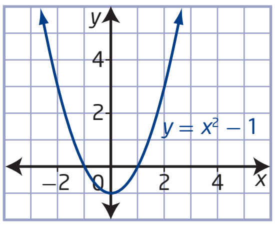
3. State whether or not the graph of the relation is a function. Then, use the horizontal line test to determine whether the inverse relation will be a function.

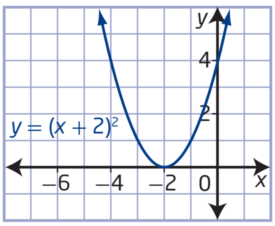
a) 

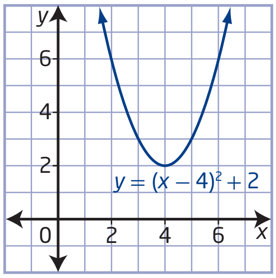
b) 

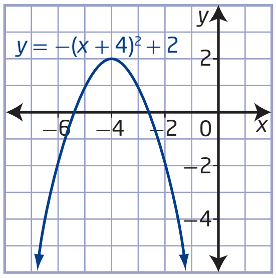
c) 

4. For each graph, identify a restricted domain for which the function has an inverse that is also a function.

a) 

b) 

c) 

d) 

5. Algebraically determine the equation of the inverse of each function.

a) $f(x) = 7x$

b) $f(x) = -3x + 4$

c) $f(x) = \dfrac{x + 4}{3}$

d) $f(x) = \dfrac{x}{3} - 5$

e) $f(x) = 5 - 2x$

f) $f(x) = \dfrac{1}{2}(x + 6)$

6. Match the function with its inverse.

Function

a) $y = 2x + 5$

b) $y = \dfrac{1}{2}x - 4$

c) $y = 6 - 3x$

d) $y = x^2 - 12, x \geq 0$

e) $y = \dfrac{1}{2}(x + 1)^2, x \leq -1$

Inverse

A $y = \sqrt{x + 12}$

B $y = \dfrac{6 - x}{3}$

C $y = 2x + 8$

D $y = -\sqrt{2x} - 1$

E $y = \dfrac{x - 5}{2}$

Apply

7. For each table, plot the ordered pairs (x, y) and the ordered pairs (y, x). State the domain of the function and its inverse.

a)

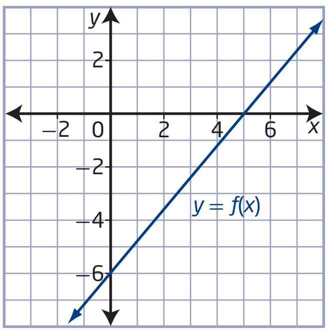
x	y
-2	-2
-1	1
0	4
1	7
2	10

b)

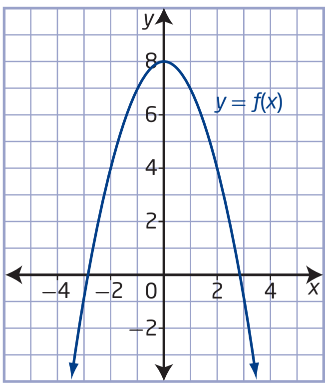
x	y
-6	2
-4	4
-1	5
2	5
5	3

8. Copy each graph of $y = f(x)$ and then sketch the graph of its inverse. Determine if the inverse is a function. Give a reason for your answer.

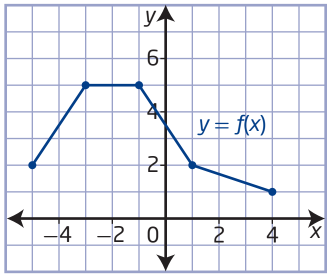
a)



b)



c)



9. For each of the following functions,
- determine the equation for the inverse, $f^{-1}(x)$
- graph $f(x)$ and $f^{-1}(x)$
- determine the domain and range of $f(x)$ and $f^{-1}(x)$

a) $f(x) = 3x + 2$

b) $f(x) = 4 - 2x$

c) $f(x) = \frac{1}{2}x - 6$

d) $f(x) = x^2 + 2,\ x \le 0$

e) $f(x) = 2 - x^2,\ x \ge 0$

10. For each function $f(x)$,

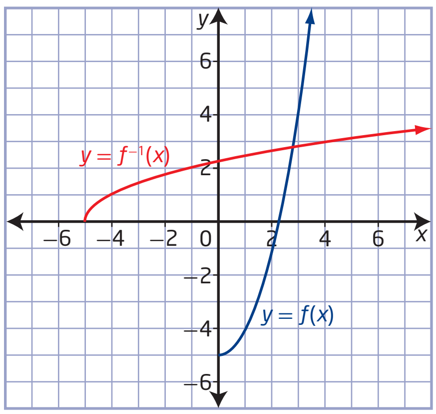
i) determine the equation of the inverse of $f(x)$ by first rewriting the function in the form $y = a(x - h)^2 + k$

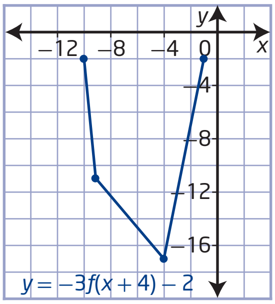
ii) graph $f(x)$ and the inverse of $f(x)$

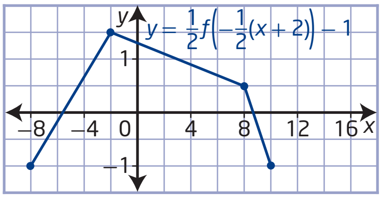
a) $f(x) = x^2 + 8x + 12$

b) $f(x) = x^2 - 4x + 2$

11. Jocelyn and Gerry determine that the inverse of the function $f(x) = x^2 - 5,\ x \ge 0$, is $f^{-1}(x) = \sqrt{x + 5}$. Does the graph verify that these functions are inverses of each other? Explain why.

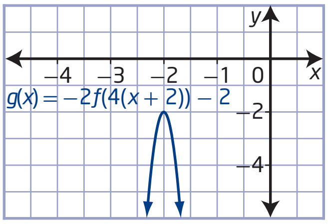


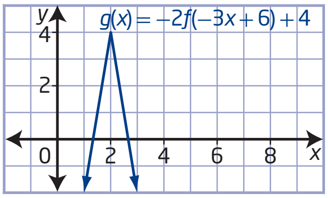
e) 

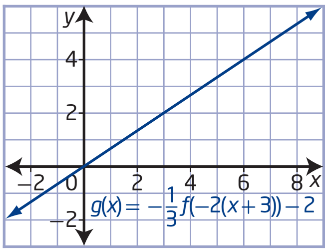
f) 

10. a) $y = -3f(x - 8) + 10$ **b)** $y = -2f(x - 3) + 2$

c) $y = -\frac{1}{2}f(-2(x + 4)) + 7$

11. a) 

b) 

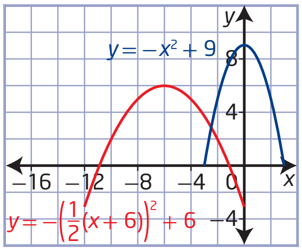
c) 

12. a) A′(−11, −2), B′(−7, 6), C′(−3, 4), D′(−1, 5), E′(3, −2)

b) $y = -f\left(\frac{1}{2}(x + 3)\right) + 4$

13. a) The graphs are in two locations because the transformations performed to obtain Graph 2 do not match those in $y = |2x - 6| + 2$. Gil forgot to factor out the coefficient of the x-term, 2, from −6. The horizontal translation should have been 3 units right, not 6 units.

b) He should have rewritten the function as $y = |2(x - 3)| + 2$.

14. a) 

b) $y = -\left(\frac{1}{2}(x + 6)\right)^2 + 6$

15. a) $(-a, 0)$, $(0, -b)$ **b)** $(2a, 0)$, $(0, 2b)$

c) and **d)** There is not enough information to determine the locations of the new intercepts. When a transformation involves translations, the locations of the new intercepts will vary with different base functions.

16. a) $A = -2x^3 + 18x$ **b)** $A = -\frac{1}{8}x^3 + 18x$

c) For (2, 5), the area of the rectangle in part a) is 20 square units.

$A = -2x^3 + 18x$

$A = -2(2)^3 + 18(2)$

$A = 20$

For (8, 5), the area of the rectangle in part b) is 80 square units.

$A = -\frac{1}{8}x^3 + 18x$

$A = -\frac{1}{8}(8)^3 + 18(8)$

$A = 80$

17. $y = 36(x - 2)^2 + 6(x - 2) - 2$

18. Example: vertical stretches and horizontal stretches followed by reflections

C1 Step 1 They are reflections in the axes.
1: $y = x + 3$, 2: $y = -x - 3$, 3: $y = x - 3$

Step 2 They are vertical translations coupled with reflections. 1: $y = x^2 + 1$, 2: $y = x^2 - 1$, 3: $y = -x^2$, 4: $y = -x^2 - 1$

C2 a) The cost of making $b + 12$ bracelets, and it is a horizontal translation.

b) The cost of making b bracelets plus 12 more dollars, and it is a vertical translation.

c) Triple the cost of making b bracelets, and it is a vertical stretch.

d) The cost of making $\frac{b}{2}$ bracelets, and it is a horizontal stretch.

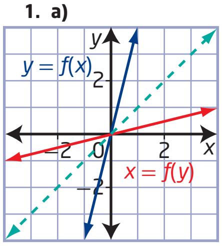
C3 $y = 2(x - 3)^2 + 1$; a vertical stretch by a factor of 2 and a translation of 3 units right and 1 unit up

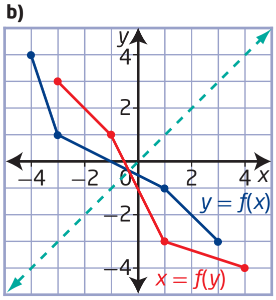
C4 a) H is repeated; J is transposed; K is repeated and transposed

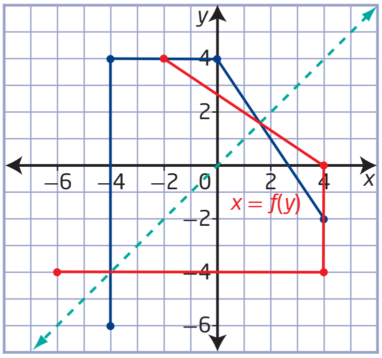
b) H is in retrograde; J is inverted; K is in retrograde and inverted

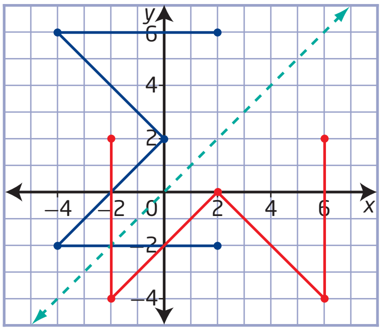
c) H is inverted, repeated, and transposed; J is in retrograde inversion and repeated; K is in retrograde and transposed

1.4 Inverse of a Relation, pages 51 to 55

1. a) 

b) 

2. a) 

b) 

3. a) The graph is a function but the inverse will be a relation.

b) The graph and its inverse are functions.

c) The graph and its inverse are relations.

4. Examples:

a) $\{x \mid x \geq 0, x \in \mathrm{R}\}$ or $\{x \mid x \leq 0, x \in \mathrm{R}\}$

b) $\{x \mid x \geq -2, x \in \mathrm{R}\}$ or $\{x \mid x \leq -2, x \in \mathrm{R}\}$

c) $\{x \mid x \geq 4, x \in \mathrm{R}\}$ or $\{x \mid x \leq 4, x \in \mathrm{R}\}$

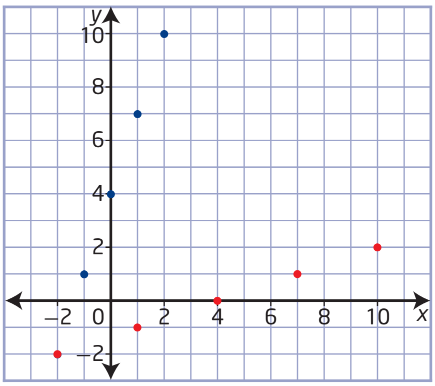
d) $\{x \mid x \geq -4, x \in \mathrm{R}\}$ or $\{x \mid x \leq -4, x \in \mathrm{R}\}$

5. a) $f^{-1}(x) = \frac{1}{7}x$ **b)** $f^{-1}(x) = -\frac{1}{3}(x - 4)$

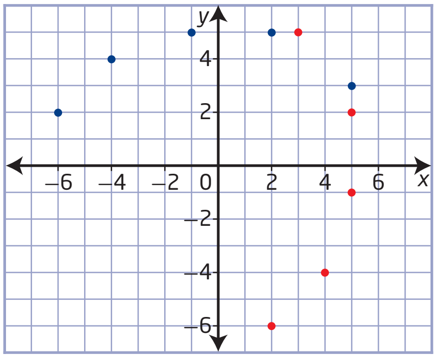
c) $f^{-1}(x) = 3x - 4$ **d)** $f^{-1}(x) = 3x + 15$

e) $f^{-1}(x) = -\frac{1}{2}(x - 5)$ **f)** $f^{-1}(x) = 2x - 6$

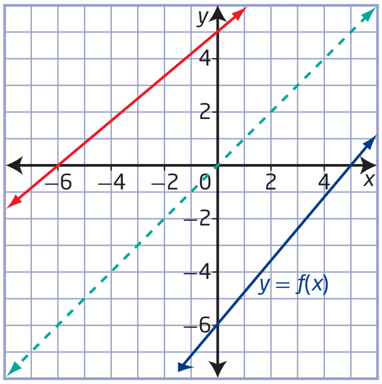
6. a) E **b)** C **c)** B **d)** A **e)** D

7. a) 

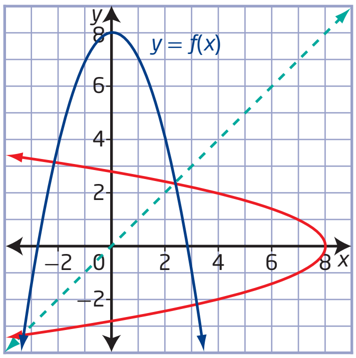
function: domain $\{-2, -1, 0, 1, 2\}$, range $\{-2, 1, 4, 7, 10\}$
inverse: domain $\{-2, 1, 4, 7, 10\}$, range $\{-2, -1, 0, 1, 2\}$

b) 

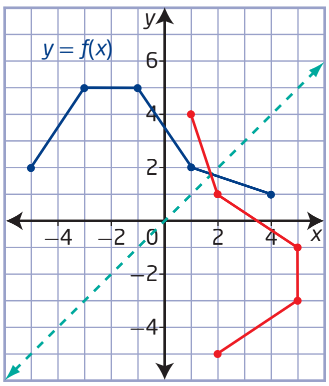
function: domain $\{-6, -4, -1, 2, 5\}$, range $\{2, 3, 4, 5\}$
inverse: domain $\{2, 3, 4, 5\}$, range $\{-6, -4, -1, 2, 5\}$

8. a) 

The inverse is a function; it passes the vertical line test.

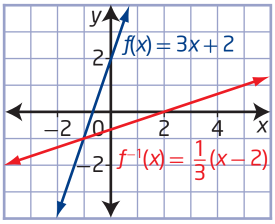
b) 

The inverse is not a function; it does not pass the vertical line test.

c) 

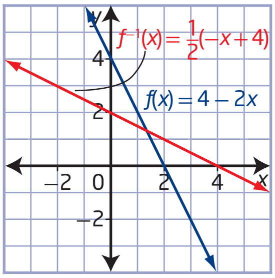
The inverse is not a function; it does not pass the vertical line test.

9. a) $f^{-1}(x) = \frac{1}{3}(x - 2)$



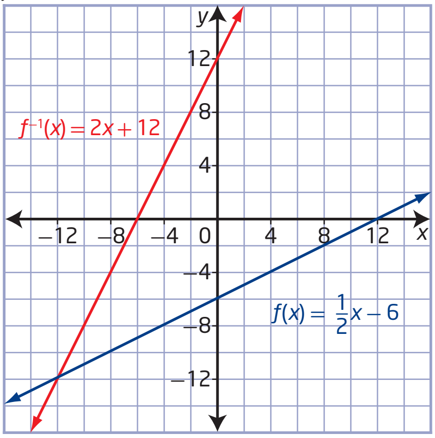
$f(x)$:
domain $\{x \mid x \in \mathrm{R}\}$,
range $\{y \mid y \in \mathrm{R}\}$
$f^{-1}(x)$:
domain $\{x \mid x \in \mathrm{R}\}$,
range $\{y \mid y \in \mathrm{R}\}$

b) $f^{-1}(x) = \frac{1}{2}(-x + 4)$



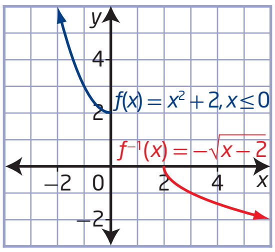
$f(x)$:
domain $\{x \mid x \in \mathrm{R}\}$,
range $\{y \mid y \in \mathrm{R}\}$
$f^{-1}(x)$:
domain $\{x \mid x \in \mathrm{R}\}$,
range $\{y \mid y \in \mathrm{R}\}$

c) $f^{-1}(x) = 2x + 12$



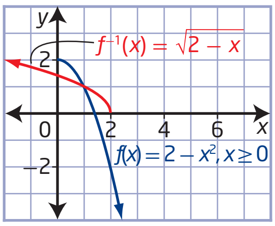
$f(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$
$f^{-1}(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

d) $f^{-1}(x) = -\sqrt{x - 2}$



$f(x)$: domain $\{x \mid x \le 0, x \in \mathrm{R}\}$, range $\{y \mid y \ge 2, y \in \mathrm{R}\}$
$f^{-1}(x)$: domain $\{x \mid x \ge 2, x \in \mathrm{R}\}$, range $\{y \mid y \le 0, y \in \mathrm{R}\}$

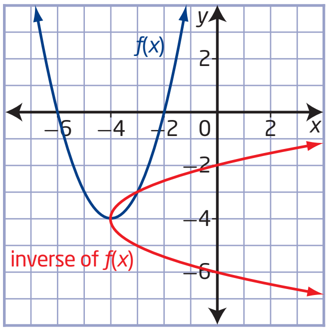
e) $f^{-1}(x) = \sqrt{2 - x}$



$f(x)$: domain $\{x \mid x \ge 0, x \in \mathrm{R}\}$, range $\{y \mid y \le 2, y \in \mathrm{R}\}$
$f^{-1}(x)$: domain $\{x \mid x \le 2, x \in \mathrm{R}\}$, range $\{y \mid y \ge 0, y \in \mathrm{R}\}$

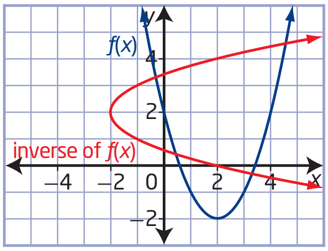
10. a) i) $f(x) = (x + 4)^2 - 4$, inverse of $f(x) = \pm\sqrt{x + 4} - 4$

ii)



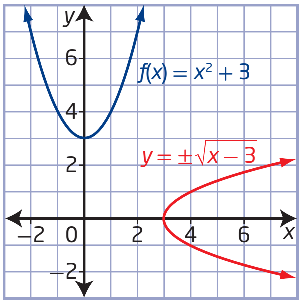
b) i) $y = (x - 2)^2 - 2$, $y = \pm\sqrt{x + 2} + 2$

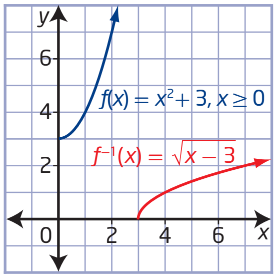
ii)



11. Yes, the graphs are reflections of each other in the line $y = x$.

12. a) $y = \pm\sqrt{x - 3}$ restricted domain $\{x \mid x \ge 0, x \in \mathrm{R}\}$





b) $y = \pm\sqrt{2x}$ restricted domain $\{x \mid x \ge 0, x \in \mathrm{R}\}$