Check Your Understanding

Practise

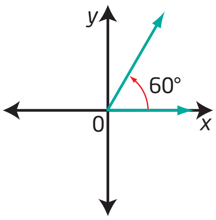
1. Sketch an angle in standard position so that the terminal arm passes through each point.

a) (2, 6) **b)** (−4, 2)

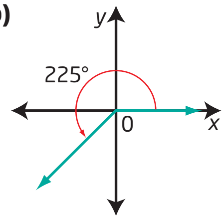
c) (−5, −2) **d)** (−1, 0)

2. Determine the exact values of the sine, cosine, and tangent ratios for each angle.

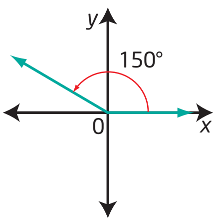
a)



b)



c)



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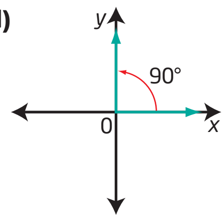
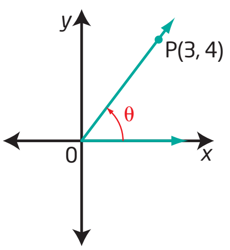
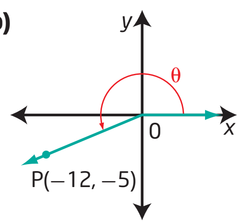


3. The coordinates of a point P on the terminal arm of each angle are shown. Write the exact trigonometric ratios sin θ, cos θ, and tan θ for each.

a)



b)



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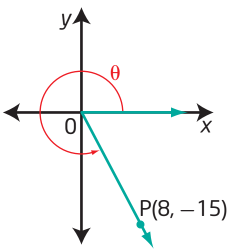
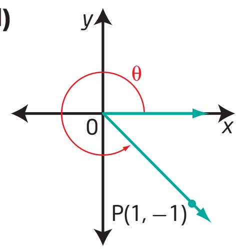


d)



4. For each description, in which quadrant does the terminal arm of angle θ lie?

a) $\cos\theta < 0$ and $\sin\theta > 0$

b) $\cos\theta > 0$ and $\tan\theta > 0$

c) $\sin\theta < 0$ and $\cos\theta < 0$

d) $\tan\theta < 0$ and $\cos\theta > 0$

5. Determine the exact values of sin θ, cos θ, and tan θ if the terminal arm of an angle in standard position passes through the given point.

a) P(−5, 12)

b) P(5, −3)

c) P(6, 3)

d) P(−24, −10)

6. Without using a calculator, state whether each ratio is positive or negative.

a) sin 155°

b) cos 320°

c) tan 120°

d) cos 220°

7. An angle is in standard position such that $\sin\theta = \frac{5}{13}$.

a) Sketch a diagram to show the two possible positions of the angle.

b) Determine the possible values of θ, to the nearest degree, if $0° \le \theta < 360°$.

8. An angle in standard position has its terminal arm in the stated quadrant. Determine the exact values for the other two primary trigonometric ratios for each.

	Ratio Value	Quadrant
a)	$\cos\theta = -\frac{2}{3}$	II
b)	$\sin\theta = \frac{3}{5}$	I
c)	$\tan\theta = -\frac{4}{5}$	IV
d)	$\sin\theta = -\frac{1}{3}$	III
e)	$\tan\theta = 1$	III

9. Solve each equation, for $0° \leq \theta < 360°$, using a diagram involving a special right triangle.

a) $\cos \theta = \frac{1}{2}$

b) $\cos \theta = -\frac{1}{\sqrt{2}}$

c) $\tan \theta = -\frac{1}{\sqrt{3}}$

d) $\sin \theta = -\frac{\sqrt{3}}{2}$

e) $\tan \theta = \sqrt{3}$

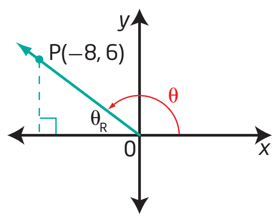
f) $\tan \theta = -1$

10. Copy and complete the table using the coordinates of a point on the terminal arm.

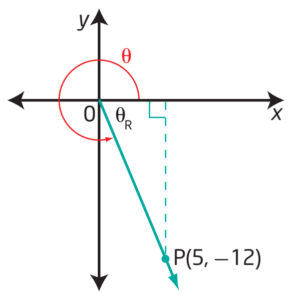
θ	sin θ	cos θ	tan θ
0°			
90°			
180°			
270°			
360°			

11. Determine the values of x, y, r, $\sin \theta$, $\cos \theta$, and $\tan \theta$ in each.

a)



b)



Apply

12. Point P(−9, 4) is on the terminal arm of an angle θ.

a) Sketch the angle in standard position.

b) What is the measure of the reference angle, to the nearest degree?

c) What is the measure of θ, to the nearest degree?

13. Point P(7, −24) is on the terminal arm of an angle, θ.

a) Sketch the angle in standard position.

b) What is the measure of the reference angle, to the nearest degree?

c) What is the measure of θ, to the nearest degree?

14. a) Determine sin θ when the terminal arm of an angle in standard position passes through the point P(2, 4).

b) Extend the terminal arm to include the point Q(4, 8). Determine sin θ for the angle in standard position whose terminal arm passes through point Q.

c) Extend the terminal arm to include the point R(8, 16). Determine sin θ for the angle in standard position whose terminal arm passes through point R.

d) Explain your results from parts a), b), and c). What do you notice? Why does this happen?

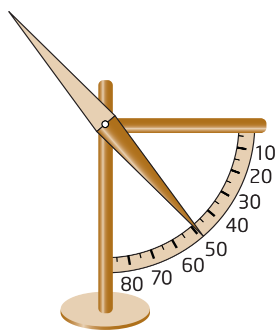
15. The point P(k, 24) is 25 units from the origin. If P lies on the terminal arm of an angle, θ, in standard position, $0° \leq \theta < 360°$, determine

a) the measure(s) of θ

b) the sine, cosine, and tangent ratios for θ

16. If $\cos \theta = \frac{1}{5}$ and $\tan \theta = 2\sqrt{6}$, determine the exact value of sin θ.

17. The angle between the horizontal and Earth's magnetic field is called the angle of dip. Some migratory birds may be capable of detecting changes in the angle of dip, which helps them navigate. The angle of dip at the magnetic equator is 0°, while the angle at the North and South Poles is 90°. Determine the exact values of sin θ, cos θ, and tan θ for the angles of dip at the magnetic equator and the North and South Poles.



22. $x^2 + y^2 = r^2$

23. a)

θ	20°	40°	60°	80°
sin θ	0.3420	0.6428	0.8660	0.9848
sin (180° − θ)	0.3420	0.6428	0.8660	0.9848
sin (180° + θ)	−0.3420	−0.6428	−0.8660	−0.9848
sin (360° − θ)	−0.3420	−0.6428	−0.8660	−0.9848

b) Each angle in standard position has the same reference angle, but the sine ratio differs in sign based on the quadrant location. The sine ratio is positive in quadrants I and II and negative in quadrants III and IV.

c) The ratios would be the same as those for the reference angle for cos θ and tan θ in quadrant I but may have different signs than sin θ in each of the other quadrants.

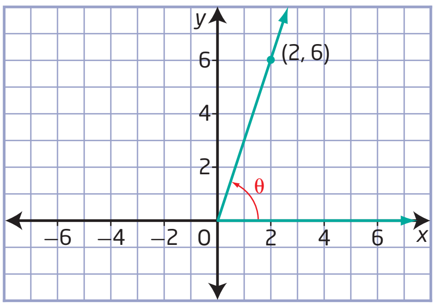
24. a) $\frac{3025\sqrt{3}}{16}$ ft

b) As the angle increases to 45° the distance increases and then decreases after 45°.

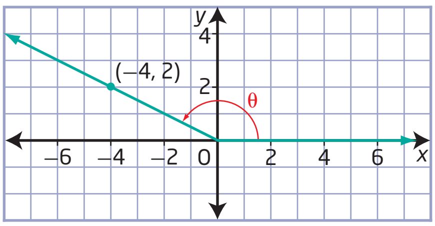
c) The greatest distance occurs with an angle of 45°. The product of cos θ and sin θ has a maximum value when θ = 45°.

2.2 Trigonometric Ratios of Any Angle, pages 96 to 99

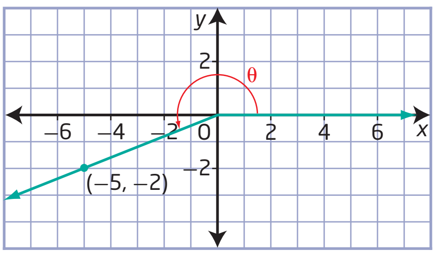
1. a)



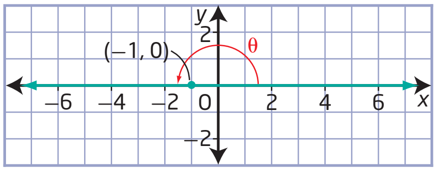
b)



c)



d)



2. a) $\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$, $\tan 60° = \sqrt{3}$

b) $\sin 225° = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$,

$\cos 225° = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$, $\tan 225° = 1$

c) $\sin 150° = \frac{1}{2}$, $\cos 150° = -\frac{\sqrt{3}}{2}$,

$\tan 150° = -\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$

d) sin 90° = 1, cos 90° = 0, tan 90° is undefined

3. a) $\sin \theta = \frac{4}{5}$, $\cos \theta = \frac{3}{5}$, $\tan \theta = \frac{4}{3}$

b) $\sin \theta = -\frac{5}{13}$, $\cos \theta = -\frac{12}{13}$, $\tan \theta = \frac{5}{12}$

c) $\sin \theta = -\frac{15}{17}$, $\cos \theta = \frac{8}{17}$, $\tan \theta = -\frac{15}{8}$

d) $\sin \theta = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$, $\cos \theta = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$,

$\tan \theta = -1$

4. a) II **b)** I **c)** III **d)** IV

5. a) $\sin \theta = \frac{12}{13}$, $\cos \theta = -\frac{5}{13}$, $\tan \theta = -\frac{12}{5}$

b) $\sin \theta = -\frac{3}{\sqrt{34}}$ or $-\frac{3\sqrt{34}}{34}$,

$\cos \theta = \frac{5}{\sqrt{34}}$ or $\frac{5\sqrt{34}}{34}$, $\tan \theta = -\frac{3}{5}$

c) $\sin \theta = \frac{3}{\sqrt{45}}$ or $\frac{1}{\sqrt{5}}$, $\cos \theta = \frac{6}{\sqrt{45}}$ or $\frac{2}{\sqrt{5}}$,

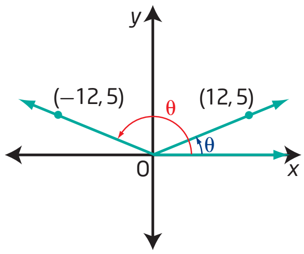
$\tan \theta = \frac{1}{2}$

d) $\sin \theta = -\frac{5}{13}$, $\cos \theta = -\frac{12}{13}$, $\tan \theta = \frac{5}{12}$

6. a) positive **b)** positive

c) negative **d)** negative

7. a)



b) 23° or 157°

8. a) $\sin \theta = \frac{\sqrt{5}}{3}$, $\tan \theta = -\frac{\sqrt{5}}{2}$

b) $\cos \theta = \frac{4}{5}$, $\tan \theta = \frac{3}{4}$

c) $\sin \theta = -\frac{4}{\sqrt{41}}$ or $-\frac{4\sqrt{41}}{41}$,

$\cos \theta = \frac{5}{\sqrt{41}}$ or $\frac{5\sqrt{41}}{41}$

d) $\cos \theta = -\frac{2\sqrt{2}}{3}$, $\tan \theta = \frac{\sqrt{2}}{4}$

e) $\sin \theta = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$,

$\cos \theta = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$

9. a) 60° and 300° **b)** 135° and 225°

c) 150° and 330° **d)** 240° and 300°

e) 60° and 240° **f)** 135° and 315°

10.

θ	sin θ	cos θ	tan θ
0°	0	1	0
90°	1	0	undefined
180°	0	−1	0
270°	−1	0	undefined
360°	0	1	0

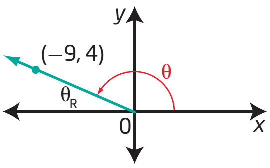
11. a) $x = -8$, $y = 6$, $r = 10$, $\sin \theta = \frac{3}{5}$,

$\cos \theta = -\frac{4}{5}$, $\tan \theta = -\frac{3}{4}$

b) $x = 5$, $y = -12$, $r = 13$, $\sin \theta = -\frac{12}{13}$,

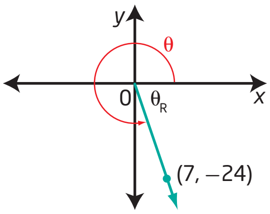
$\cos \theta = \frac{5}{13}$, $\tan \theta = -\frac{12}{5}$

12. a)



b) 24° **c)** 156°

13. a)



b) 74° **c)** 286°

14. a) $\sin \theta = \frac{2}{\sqrt{5}}$ or $\frac{2\sqrt{5}}{5}$

b) $\sin \theta = \frac{2}{\sqrt{5}}$ or $\frac{2\sqrt{5}}{5}$

c) $\sin \theta = \frac{2}{\sqrt{5}}$ or $\frac{2\sqrt{5}}{5}$

d) They all have the same sine ratio. This happens because the points P, Q, and R are collinear. They are on the same terminal arm.

15. a) 74° and 106°

b) $\sin \theta = \frac{24}{25}$, $\cos \theta = \pm\frac{7}{25}$, $\tan \theta = \pm\frac{24}{7}$

16. $\sin \theta = \frac{2\sqrt{6}}{5}$

17. $\sin 0° = 0$, $\cos 0° = 1$, $\tan 0° = 0$, $\sin 90° = 1$, $\cos 90° = 0$, $\tan 90°$ is undefined

18. a) True. θ_R for 151° is 29° and is in quadrant II. The sine ratio is positive in quadrants I and II.

b) True; both sin 225° and cos 135° have a reference angle of 45° and $\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$.

c) False; tan 135° is in quadrant II, where $\tan \theta < 0$, and tan 225° is in quadrant III, where $\tan \theta > 0$.

d) True; from the reference angles in a 30°-60°-90° triangle, $\sin 60° = \cos 330° = \frac{\sqrt{3}}{2}$.

e) True; the terminal arms lie on the axes, passing through P(0, −1) and P(−1, 0), respectively, so sin 270° = cos 180° = −1.

19.

θ	sin θ	cos θ	tan θ
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
45°	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	undefined
120°	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
135°	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	−1
150°	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$
180°	0	−1	0
210°	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
225°	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	1
240°	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$
270°	−1	0	undefined
300°	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$
315°	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	−1
330°	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$
360°	0	1	0

20. a) ∠A = 45°, ∠B = 135°, ∠C = 225°, ∠D = 315°

b) $A\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, $B\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, $C\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$, $D\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$