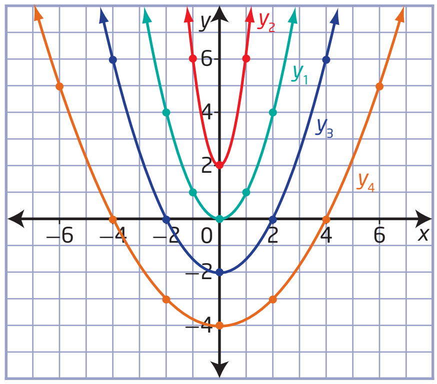
Check Your Understanding

Practise

1. Describe how you can obtain the graph of each function from the graph of $f(x) = x^2$. State the direction of opening, whether it has a maximum or a minimum value, and the range for each.
   a) $f(x) = 7x^2$
   b) $f(x) = \frac{1}{6}x^2$
   c) $f(x) = -4x^2$
   d) $f(x) = -0.2x^2$
2. Describe how the graphs of the functions in each pair are related. Then, sketch the graph of the second function in each pair, and determine the vertex, the equation of the axis of symmetry, the domain and range, and any intercepts.
   a) $y = x^2$ and $y = x^2 + 1$
   b) $y = x^2$ and $y = (x - 2)^2$
   c) $y = x^2$ and $y = x^2 - 4$
   d) $y = x^2$ and $y = (x + 3)^2$
3. Describe how to sketch the graph of each function using transformations.
   a) $f(x) = (x + 5)^2 + 11$
   b) $f(x) = -3x^2 - 10$
   c) $f(x) = 5(x + 20)^2 - 21$
   d) $f(x) = -\frac{1}{8}(x - 5.6)^2 + 13.8$
4. Sketch the graph of each function. Identify the vertex, the axis of symmetry, the direction of opening, the maximum or minimum value, the domain and range, and any intercepts.
   a) $y = -(x - 3)^2 + 9$
   b) $y = 0.25(x + 4)^2 + 1$
   c) $y = -3(x - 1)^2 + 12$
   d) $y = \frac{1}{2}(x - 2)^2 - 2$
5. **a)** Write a quadratic function in vertex form for each parabola in the graph.



   b) Suppose four new parabolas open downward instead of upward but have the same shape and vertex as each parabola in the graph. Write a quadratic function in vertex form for each new parabola.
   c) Write the quadratic functions in vertex form of four parabolas that are identical to the four in the graph but translated 4 units to the left.
   d) Suppose the four parabolas in the graph are translated 2 units down. Write a quadratic function in vertex form for each new parabola.
6. For the function $f(x) = 5(x - 15)^2 - 100$, explain how you can identify each of the following without graphing.
   a) the coordinates of the vertex
   b) the equation of the axis of symmetry
   c) the direction of opening
   d) whether the function has a maximum or minimum value, and what that value is
   e) the domain and range
   f) the number of x-intercepts

7. Without graphing, identify the location of the vertex and the axis of symmetry, the direction of opening and the maximum or minimum value, the domain and range, and the number of x-intercepts for each function.

a) $y = -4x^2 + 14$

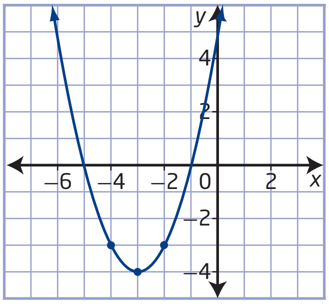
b) $y = (x + 18)^2 - 8$

c) $y = 6(x - 7)^2$

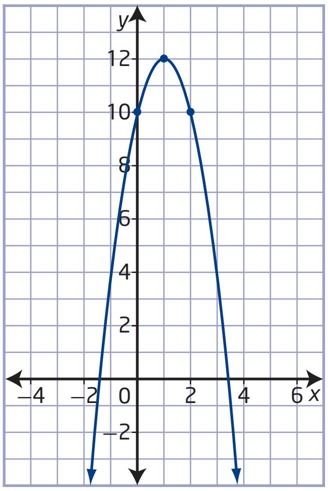
d) $y = -\frac{1}{9}(x + 4)^2 - 36$

8. Determine the quadratic function in vertex form for each parabola.

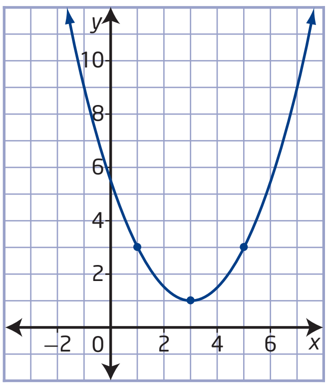
a)



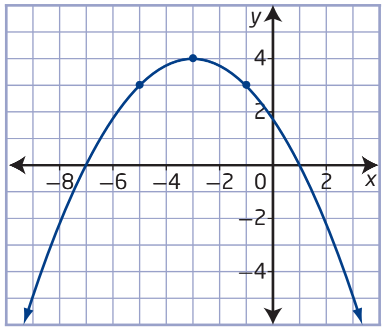
b)



c)



d)



9. Determine a quadratic function in vertex form that has the given characteristics.

a) vertex at (0, 0), passing through the point (6, −9)

b) vertex at (0, −6), passing through the point (3, 21)

c) vertex at (2, 5), passing through the point (4, −11)

d) vertex at (−3, −10), passing through the point (2, −5)

Apply

10. The point (4, 16) is on the graph of $f(x) = x^2$. Describe what happens to the point when each of the following sets of transformations is performed in the order listed. Identify the corresponding point on the transformed graph.

a) a horizontal translation of 5 units to the left and then a vertical translation of 8 units up

b) a multiplication of the y-values by a factor of $\frac{1}{4}$ and then a reflection in the x-axis

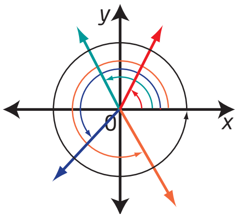
c) a reflection in the x-axis and then a horizontal translation of 10 units to the right

d) a multiplication of the y-values by a factor of 3 and then a vertical translation of 8 units down

11. Describe how to obtain the graph of $y = 20 - 5x^2$ using transformations on the graph of $y = x^2$.

12. Quadratic functions do not all have the same number of x-intercepts. Is the same true about y-intercepts? Explain.

4. C
5. D
6. \$0.15 per cup
7. 45°
8. 300°
9. 2775
10. a) 5 b) −6
c) $t_n = 5n - 11$ d) $S_{10} = 165$
11. \$14 880.35
12. 4 km
13. a) 64, 32, 16, 8, … b) $t_n = 64\left(\frac{1}{2}\right)^{n-1}$
c) 63 games
14. a)

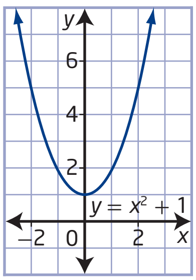


b) 60, 120, 180, 240, 300, 360
c) $t_n = 60n$
15. a) 58° b) 5.3 m
16. 38°

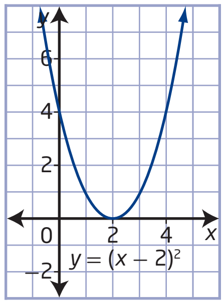
Chapter 3 Quadratic Functions

3.1 Investigating Quadratic Functions in Vertex Form, pages 157 to 162

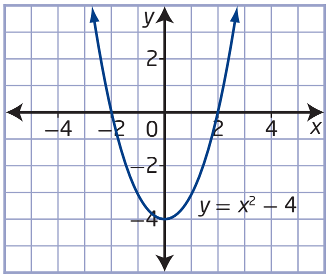
1. a) Since $a > 0$ in $f(x) = 7x^2$, the graph opens upward, has a minimum value, and has a range of $\{y \mid y \geq 0, y \in R\}$.
b) Since $a > 0$ in $f(x) = \frac{1}{6}x^2$, the graph opens upward, has a minimum value, and has a range of $\{y \mid y \geq 0, y \in R\}$.
c) Since $a < 0$ in $f(x) = -4x^2$, the graph opens downward, has a maximum value, and has a range of $\{y \mid y \leq 0, y \in R\}$.
d) Since $a < 0$ in $f(x) = -0.2x^2$, the graph opens downward, has a maximum value, and has a range of $\{y \mid y \leq 0, y \in R\}$.
2. a) The shapes of the graphs are the same with the parabola of $y = x^2 + 1$ being one unit higher. vertex: (0, 1), axis of symmetry: $x = 0$, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \geq 1, y \in R\}$, no x-intercepts, y-intercept occurs at (0, 1)



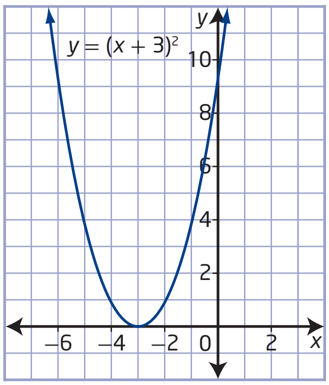
b) The shapes of the graphs are the same with the parabola of $y = (x - 2)^2$ being two units to the right. vertex: (2, 0), axis of symmetry: $x = 2$, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \geq 0, y \in R\}$, x-intercept occurs at (2, 0), y-intercept occurs at (0, 4)



c) The shapes of the graphs are the same with the parabola of $y = x^2 - 4$ being four units lower.



vertex: (0, −4), axis of symmetry: $x = 0$, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \geq -4, y \in R\}$, x-intercepts occur at (−2, 0) and (2, 0), y-intercept occurs at (0, −4)
d) The shapes of the graphs are the same with the parabola of $y = (x + 3)^2$ being three units to the left.



vertex: (−3, 0), axis of symmetry: $x = -3$, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \geq 0, y \in R\}$, x-intercept occurs at (−3, 0), y-intercept occurs at (0, 9)

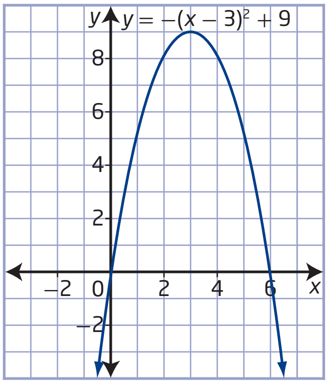
3. a) Given the graph of $y = x^2$, move the entire graph 5 units to the left and 11 units up.

b) Given the graph of $y = x^2$, apply the change in width, which is a multiplication of the y-values by a factor of 3, making it narrower, reflect it in the x-axis so it opens downward, and move the entire new graph down 10 units.

c) Given the graph of $y = x^2$, apply the change in width, which is a multiplication of the y-values by a factor of 5, making it narrower. Move the entire new graph 20 units to the left and 21 units down.

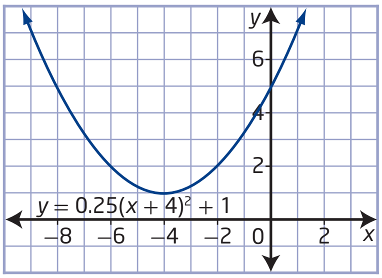
d) Given the graph of $y = x^2$, apply the change in width, which is a multiplication of the y-values by a factor of $\frac{1}{8}$, making it wider, reflect it in the x-axis so it opens downward, and move the entire new graph 5.6 units to the right and 13.8 units up.

4. a)



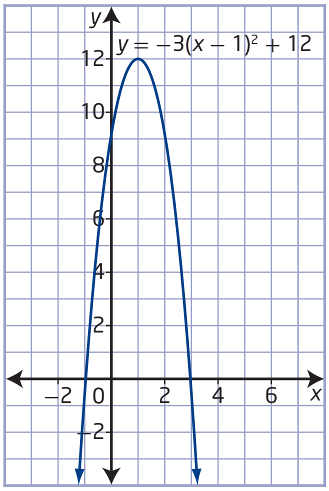
vertex: (3, 9), axis of symmetry: $x = 3$, opens downward, maximum value of 9, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \leq 9, y \in \mathrm{R}\}$, x-intercepts occur at (0, 0) and (6, 0), y-intercept occurs at (0, 0)

b)



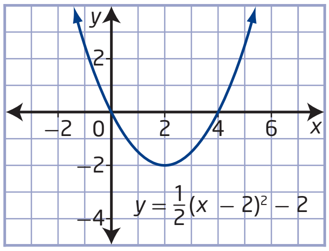
vertex: (–4, 1), axis of symmetry: $x = -4$, opens upward, minimum value of 1, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq 1, y \in \mathrm{R}\}$, no x-intercepts, y-intercept occurs at (0, 5)

c)



vertex: (1, 12), axis of symmetry: $x = 1$, opens downward, maximum value of 12, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \leq 12, y \in \mathrm{R}\}$, x-intercepts occur at (–1, 0) and (3, 0), y-intercept occurs at (0, 9)

d)



vertex: (2, –2), axis of symmetry: $x = 2$, opens upward, minimum value of –2, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq -2, y \in \mathrm{R}\}$, x-intercepts occur at (0, 0) and (4, 0), y-intercept occurs at (0, 0)

5. a) $y_1 = x^2$, $y_2 = 4x^2 + 2$, $y_3 = \frac{1}{2}x^2 - 2$, $y_4 = \frac{1}{4}x^2 - 4$

b) $y_1 = -x^2$, $y_2 = -4x^2 + 2$, $y_3 = -\frac{1}{2}x^2 - 2$, $y_4 = -\frac{1}{4}x^2 - 4$

c) $y_1 = (x + 4)^2$, $y_2 = 4(x + 4)^2 + 2$, $y_3 = \frac{1}{2}(x + 4)^2 - 2$, $y_4 = \frac{1}{4}(x + 4)^2 - 4$

d) $y_1 = x^2 - 2$, $y_2 = 4x^2$, $y_3 = \frac{1}{2}x^2 - 4$, $y_4 = \frac{1}{4}x^2 - 6$

6. For the function $f(x) = 5(x - 15)^2 - 100$, $a = 5$, $p = 15$, and $q = -100$.

a) The vertex is located at (p, q), or (15, –100).

b) The equation of the axis of symmetry is $x = p$, or $x = 15$.

c) Since $a > 0$, the graph opens upward.

d) Since $a > 0$, the graph has a minimum value of q, or -100.
e) The domain is $\{x \mid x \in \mathrm{R}\}$. Since the function has a minimum value of -100, the range is $\{y \mid y \geq -100, y \in \mathrm{R}\}$.
f) Since the graph has a minimum value of -100 and opens upward, there are two x-intercepts.

7. a) vertex: (0, 14), axis of symmetry: $x = 0$, opens downward, maximum value of 14, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \leq 14, y \in \mathrm{R}\}$, two x-intercepts
b) vertex: $(-18, -8)$, axis of symmetry: $x = -18$, opens upward, minimum value of -8, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq -8, y \in \mathrm{R}\}$, two x-intercepts
c) vertex: (7, 0), axis of symmetry: $x = 7$, opens upward, minimum value of 0, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$, one x-intercept
d) vertex: $(-4, -36)$, axis of symmetry: $x = -4$, opens downward, maximum value of -36, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \leq -36, y \in \mathrm{R}\}$, no x-intercepts

8. a) $y = (x + 3)^2 - 4$ **b)** $y = -2(x - 1)^2 + 12$
c) $y = \frac{1}{2}(x - 3)^2 + 1$ **d)** $y = -\frac{1}{4}(x + 3)^2 + 4$

9. a) $y = -\frac{1}{4}x^2$ **b)** $y = 3x^2 - 6$
c) $y = -4(x - 2)^2 + 5$ **d)** $y = \frac{1}{5}(x + 3)^2 - 10$

10. a) $(4, 16) \rightarrow (-1, 16) \rightarrow (-1, 24)$
b) $(4, 16) \rightarrow (4, 4) \rightarrow (4, -4)$
c) $(4, 16) \rightarrow (4, -16) \rightarrow (14, -16)$
d) $(4, 16) \rightarrow (4, 48) \rightarrow (4, 40)$

11. Starting with the graph of $y = x^2$, apply the change in width, which is a multiplication of the y-values by a factor of 5, reflect the graph in the x-axis, and then move the entire graph up 20 units.

12. Example: Quadratic functions will always have one y-intercept. Since the graphs always open upward or downward and have a domain of $\{x \mid x \in \mathrm{R}\}$, the parabola will always cross the y-axis. The graphs must always have a value at $x = 0$ and therefore have one y-intercept.

13. a) $y = \frac{1}{30}x^2$
b) The new function could be $y = \frac{1}{30}(x - 30)^2 - 30$ or $y = \frac{1}{30}(x + 30)^2 - 30$. Both graphs have the same size and shape, but the new function has been transformed by a horizontal translation of 30 units to the right or to the left and a vertical translation of 30 units down to represent a point on the edge as the origin.

14. a) The vertex is located at (36, 20 000), it opens downward, and it has a change in width by a multiplication of the y-values by a factor of 2.5 of the graph $y = x^2$. The equation of the axis of symmetry is $x = 36$, and the graph has a maximum value of 20 000.
b) 36 times
c) 20 000 people

15. Examples: If the vertex is at the origin, the quadratic function will be $y = 0.03x^2$. If the edge of the rim is at the origin, the quadratic function will be $y = 0.03(x - 20)^2 - 12$.

16. a) Example: Placing the vertex at the origin, the quadratic function is $y = \frac{1}{294}x^2$ or $y \approx 0.0034x^2$.
b) Example: If the origin is at the top of the left tower, the quadratic function is $y = \frac{1}{294}(x - 84)^2 - 24$ or $y \approx 0.0034(x - 84)^2 - 24$. If the origin is at the top of the right tower, the quadratic function is $y = \frac{1}{294}(x + 84)^2 - 24$ or $y \approx 0.0034(x + 84)^2 - 24$.
c) 8.17 m; this is the same no matter which function is used.

17. $y = -\frac{9}{121}(x - 11)^2 + 9$

18. $y = -\frac{1}{40}(x - 60)^2 + 90$

19. Example: Adding q is done after squaring the x-value, so the transformation applies directly to the parabola $y = x^2$. The value of p is added or subtracted before squaring, so the shift is opposite to the sign in the bracket to get back to the original y-value for the graph of $y = x^2$.

20. a) $y = -\frac{7}{160\,000}(x - 8000)^2 + 10\,000$
b) domain: $\{x \mid 0 \leq x \leq 16\,000, x \in \mathrm{R}\}$, range: $\{y \mid 7200 \leq y \leq 10\,000, y \in \mathrm{R}\}$

21. a) Since the vertex is located at (6, 30), $p = 6$ and $q = 30$. Substituting these values into the vertex form of a quadratic function and using the coordinates of the given point, the function is $y = -1.5(x - 6)^2 + 30$.
b) Knowing that the x-intercepts are -21 and -5, the equation of the axis of symmetry must be $x = -13$. Then, the vertex is located at $(-13, -24)$. Substituting the coordinates of the vertex and one of the x-intercepts into the vertex form, the quadratic function is $y = 0.375(x + 13)^2 - 24$.