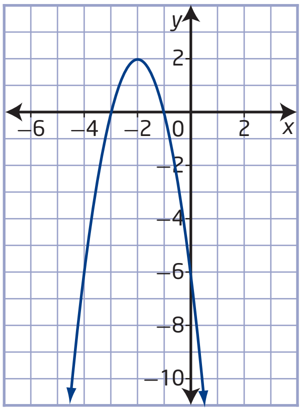
Check Your Understanding

Practise

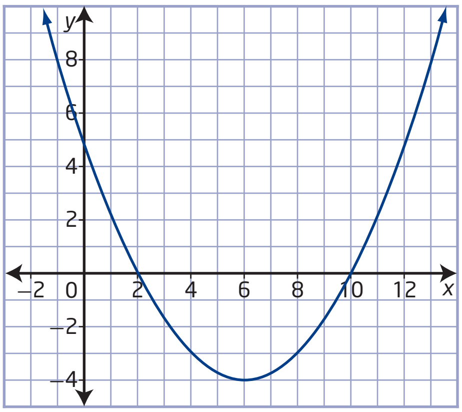
1. Which functions are quadratic? Explain.
   a) $f(x) = 2x^2 + 3x$
   b) $f(x) = 5 - 3x$
   c) $f(x) = x(x + 2)(4x - 1)$
   d) $f(x) = (2x - 5)(3x - 2)$

2. For each graph, identify the following:
   - the coordinates of the vertex
   - the equation of the axis of symmetry
   - the x-intercepts and y-intercept
   - the maximum or minimum value and how it is related to the direction of opening
   - the domain and range

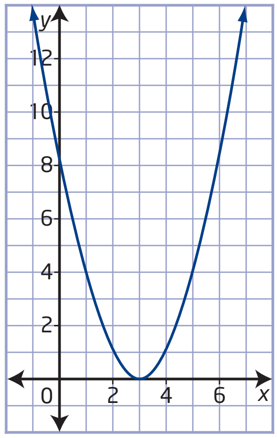
   a)



   b)



   c)



3. Show that each function fits the definition of a quadratic function by writing it in standard form.
   a) $f(x) = 5x(10 - 2x)$
   b) $f(x) = (10 - 3x)(4 - 5x)$

4. Create a table of values and then sketch the graph of each function. Determine the vertex, the axis of symmetry, the direction of opening, the maximum or minimum value, the domain and range, and any intercepts.
   a) $f(x) = x^2 - 2x - 3$
   b) $f(x) = -x^2 + 16$
   c) $p(x) = x^2 + 6x$
   d) $g(x) = -2x^2 + 8x - 10$

5. Use technology to graph each function. Identify the vertex, the axis of symmetry, the direction of opening, the maximum or minimum value, the domain and range, and any intercepts. Round values to the nearest tenth, if necessary.
   a) $y = 3x^2 + 7x - 6$
   b) $y = -2x^2 + 5x + 3$
   c) $y = 50x - 4x^2$
   d) $y = 1.2x^2 + 7.7x + 24.3$

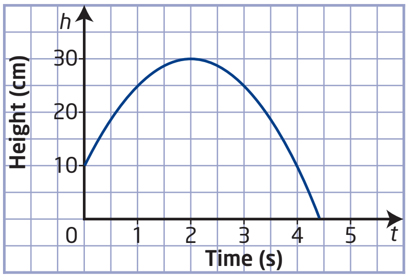
6. The x-coordinate of the vertex is given by $x = \frac{-b}{2a}$. Use this information to determine the vertex of each quadratic function.

   a) $y = x^2 + 6x + 2$

   b) $y = 3x^2 - 12x + 5$

   c) $y = -x^2 + 8x - 11$

7. A siksik, an Arctic ground squirrel, jumps from a rock, travels through the air, and then lands on the tundra. The graph shows the height of its jump as a function of time. Use the graph to answer each of the following, and identify which characteristic(s) of the graph you used in each case.



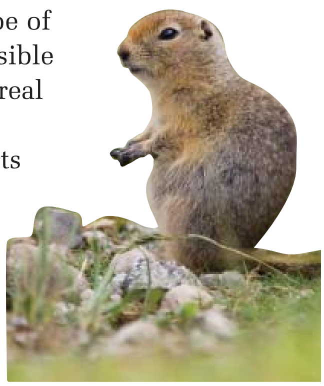
   a) What is the height of the rock that the siksik jumped from?

   b) What is the maximum height of the siksik? When did it reach that height?

   c) How long was the siksik in the air?

   d) What are the domain and range in this situation?

   e) Would this type of motion be possible for a siksik in real life? Use your answers to parts a) to d) to explain why or why not.



Did You Know?

The *siksik* is named because of the sound it makes.

8. How many x-intercepts does each function have? Explain how you know. Then, determine whether each intercept is positive, negative, or zero.

   a) a quadratic function with an axis of symmetry of $x = 0$ and a maximum value of 8

   b) a quadratic function with a vertex at $(3, 1)$, passing through the point $(1, -3)$

   c) a quadratic function with a range of $y \geq 1$

   d) a quadratic function with a y-intercept of 0 and an axis of symmetry of $x = -1$

9. Consider the function $f(x) = -16x^2 + 64x + 4$.

   a) Determine the domain and range of the function.

   b) Suppose this function represents the height, in feet, of a football kicked into the air as a function of time, in seconds. What are the domain and range in this case?

   c) Explain why the domain and range are different in parts a) and b).

Apply

10. Sketch the graph of a quadratic function that has the characteristics described in each part. Label the coordinates of three points that you know are on the curve.

   a) x-intercepts at -1 and 3 and a range of $y \geq -4$

   b) one of its x-intercepts at -5 and vertex at $(-3, -4)$

   c) axis of symmetry of $x = 1$, minimum value of 2, and passing through $(-1, 6)$

   d) vertex at $(2, 5)$ and y-intercept of 1

11. Satellite dish antennas have the shape of a parabola. Consider a satellite dish that is 80 cm across. Its cross-sectional shape can be described by the function $d(x) = 0.0125x^2 - x$, where d is the depth, in centimetres, of the dish at a horizontal distance of x centimetres from one edge of the dish.

a) What is the domain of this function?

b) Graph the function to show the cross-sectional shape of the satellite dish.

c) What is the maximum depth of the dish? Does this correspond to the maximum value of the function? Explain.

d) What is the range of the function?

e) How deep is the dish at a point 25 cm from the edge of the dish?

12. A jumping spider jumps from a log onto the ground below. Its height, h, in centimetres, as a function of time, t, in seconds, since it jumped can be modelled by the function $h(t) = -490t^2 + 75t + 12$. Where appropriate, answer the following questions to the nearest tenth.

a) Graph the function.

b) What does the h-intercept represent?

c) When does the spider reach its maximum height? What is its maximum height?

d) When does the spider land on the ground?

e) What domain and range are appropriate in this situation?

f) What is the height of the spider 0.05 s after it jumps?

Did You Know?

There are an estimated 1400 spider species in Canada. About 110 of these are jumping spiders. British Columbia has the greatest diversity of jumping spiders. Although jumping spiders are relatively small (3 mm to 10 mm in length), they can jump horizontal distances of up to 16 cm.

13. A quadratic function can model the relationship between the speed of a moving object and the wind resistance, or drag force, it experiences. For a typical car travelling on a highway, the relationship between speed and drag can be approximated with the function $f(v) = 0.002v^2$, where f is the drag force, in newtons, and v is the speed of the vehicle, in kilometres per hour.

a) What domain do you think is appropriate in this situation?

b) Considering your answer to part a), create a table of values and a graph to represent the function.

c) How can you tell from your graph that the function is not a linear function? How can you tell from your table?

d) What happens to the values of the drag force when the speed of the vehicle doubles? Does the drag force also double?

e) Why do you think a driver might be interested in understanding the relationship between the drag force and the speed of the vehicle?

Did You Know?

A *newton* (abbreviated N) is a unit of measure of force. One newton is equal to the force required to accelerate a mass of one kilogram at a rate of one metre per second squared.

22. a) Examples: I chose $x = 8$ as the axis of symmetry, I choose the position of the hoop to be (1, 10), and I allowed the basketball to be released at various heights (6 ft, 7 ft, and 8 ft) from a distance of 16 ft from the hoop. For each scenario, substitute the coordinates of the release point into the function $y = a(x - 8)^2 + q$ to get an expression for q. Then, substitute the expression for q and the coordinates of the hoop into the function. My three functions are
$y = -\frac{4}{15}(x - 8)^2 + \frac{346}{15}$,
$y = -\frac{3}{15}(x - 8)^2 + \frac{297}{15}$, and
$y = -\frac{2}{15}(x - 8)^2 + \frac{248}{15}$.

b) Example: $y = -\frac{4}{15}(x - 8)^2 + \frac{346}{15}$ ensures that the ball passes easily through the hoop.

c) domain: $\{x \mid 0 \le x \le 16, x \in \mathrm{R}\}$,
range: $\left\{y \mid 0 \le y \le \frac{346}{15}, y \in \mathrm{R}\right\}$

23. $(m + p, an + q)$

24. Examples:

a) $f(x) = -2(x - 1)^2 + 3$

b) Plot the vertex (1, 3). Determine a point on the curve, say the y-intercept, which occurs at (0, 1). Determine that the corresponding point of (0, 1) is (2, 1). Plot these two additional points and complete the sketch of the parabola.

25. Example: You can determine the number of x-intercepts if you know the location of the vertex and the direction of opening. Visualize the general position and shape of the graph based on the values of a and q. Consider $f(x) = 0.5(x + 1)^2 - 3$, $g(x) = 2(x - 3)^2$, and $h(x) = -2(x + 3)^2 - 4$. For $f(x)$, the parabola opens upward and the vertex is below the x-axis, so the graph has two x-intercepts. For $g(x)$, the parabola opens upward and the vertex is on the x-axis, so the graph has one x-intercept. For $h(x)$, the parabola opens downward and the vertex is below the x-axis, so the graph has no x-intercepts.

26. Answers may vary.

3.2 Investigating Quadratic Functions in Standard Form, pages 174 to 179

1. a) This is a quadratic function, since it is a polynomial of degree two.

b) This is not a quadratic function, since it is a polynomial of degree one.

c) This is not a quadratic function. Once the expression is expanded, it is a polynomial of degree three.

d) This is a quadratic function. Once the expression is expanded, it is a polynomial of degree two.

2. a) The coordinates of the vertex are (−2, 2). The equation of the axis of symmetry is $x = -2$. The x-intercepts occur at (−3, 0) and (−1, 0), and the y-intercept occurs at (0, −6). The graph opens downward, so the graph has a maximum of 2 of when $x = -2$. The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \le 2, y \in \mathrm{R}\}$.

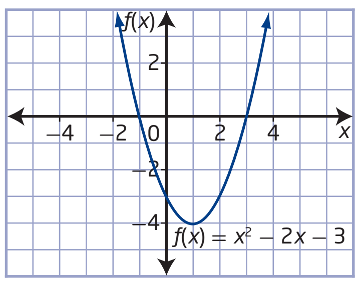
b) The coordinates of the vertex are (6, −4). The equation of the axis of symmetry is $x = 6$. The x-intercepts occur at (2, 0) and (10, 0), and the y-intercept occurs at (0, 5). The graph opens upward, so the graph has a minimum of −4 when $x = 6$. The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \ge -4, y \in \mathrm{R}\}$.

c) The coordinates of the vertex are (3, 0). The equation of the axis of symmetry is $x = 3$. The x-intercept occurs at (3, 0), and the y-intercept occurs at (0, 8). The graph opens upward, so the graph has a minimum of 0 when $x = 3$. The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \ge 0, y \in \mathrm{R}\}$.

3. a) $f(x) = -10x^2 + 50x$

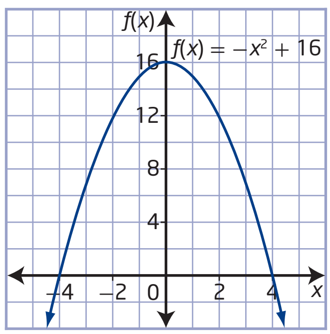
b) $f(x) = 15x^2 - 62x + 40$

4. a)



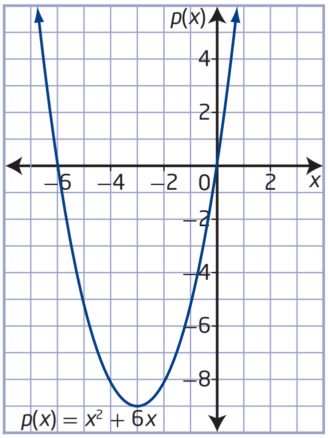
vertex is (1, −4); axis of symmetry is $x = 1$; opens upward; minimum value of −4 when $x = 1$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \ge -4, y \in \mathrm{R}\}$; x-intercepts occur at (−1, 0) and (3, 0), y-intercept occurs at (0, −3)

b)



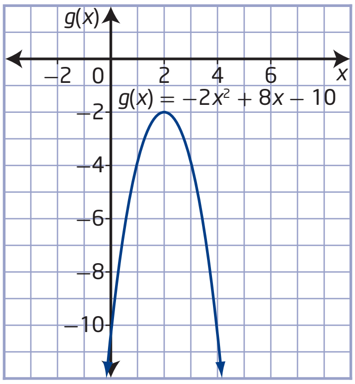
vertex is (0, 16); axis of symmetry is $x = 0$; opens downward; maximum value of 16 when $x = 0$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \le 16, y \in \mathrm{R}\}$; x-intercepts occur at $(-4, 0)$ and $(4, 0)$, y-intercept occurs at (0, 16)

c)



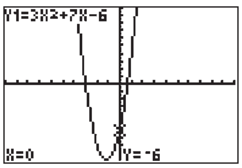
vertex is $(-3, -9)$; axis of symmetry is $x = -3$; opens upward; minimum value of -9 when $x = -3$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \ge -9, y \in \mathrm{R}\}$; x-intercepts occur at $(-6, 0)$ and (0, 0), y-intercept occurs at (0, 0)

d)



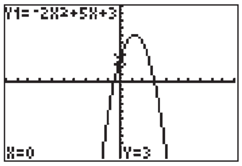
vertex is $(2, -2)$; axis of symmetry is $x = 2$; opens downward; maximum value of -2 when $x = 2$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \le -2, y \in \mathrm{R}\}$; no x-intercepts, y-intercept occurs at $(0, -10)$

5. a)



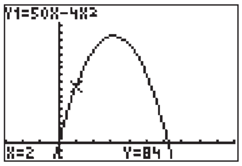
vertex is $(-1.2, -10.1)$; axis of symmetry is $x = -1.2$; opens upward; minimum value of -10.1 when $x = -1.2$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \ge -10.1, y \in \mathrm{R}\}$; x-intercepts occur at $(-3, 0)$ and (0.7, 0), y-intercept occurs at $(0, -6)$

b)



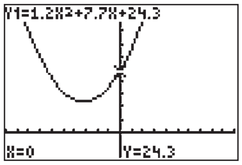
vertex is (1.3, 6.1); axis of symmetry is $x = 1.3$; opens downward; maximum value of 6.1 when $x = 1.3$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \le 6.1, y \in \mathrm{R}\}$; x-intercepts occur at $(-0.5, 0)$ and (3, 0), y-intercept occurs at (0, 3)

c)



vertex is (6.3, 156.3); axis of symmetry is $x = 6.3$; opens downward; maximum value of 156.3 when $x = 6.3$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \le 156.3, y \in \mathrm{R}\}$; x-intercepts occur at (0, 0) and (12.5, 0), y-intercept occurs at (0, 0)

d)



vertex is $(-3.2, 11.9)$; axis of symmetry is $x = -3.2$; opens upward; minimum value of 11.9 when $x = -3.2$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \ge 11.9, y \in \mathrm{R}\}$; no x-intercepts, y-intercept occurs at (0, 24.3)

6. a) $(-3, -7)$ **b)** $(2, -7)$ **c)** (4, 5)

7. a) 10 cm, h-intercept of the graph
b) 30 cm after 2 s, vertex of the parabola
c) approximately 4.4 s, t-intercept of the graph
d) domain: $\{t \mid 0 \le t \le 4.4, t \in \mathrm{R}\}$, range: $\{h \mid 0 \le h \le 30, h \in \mathrm{R}\}$
e) Example: No, siksik cannot stay in the air for 4.4 s in real life.

8. Examples:
a) Two; since the graph has a maximum value, it opens downward and would cross the x-axis at two different points. One x-intercept is negative and the other is positive.
b) Two; since the vertex is at (3, 1) and the graph passes through the point $(1, -3)$, it opens downward and crosses the x-axis at two different points. Both x-intercepts are positive.

c) Zero; since the graph has a minimum of 1 and opens upward, it will not cross the x-axis.

d) Two; since the graph has an axis of symmetry of $x = -1$ and passes through the x- and y-axes at (0, 0), the graph could open upward or downward and has another x-intercept at $(-2, 0)$. One x-intercept is zero and the other is negative.

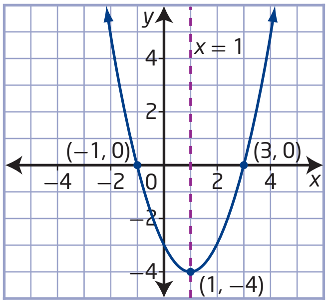
9. a) domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \leq 68, y \in \mathrm{R}\}$

b) domain: $\{x \mid 0 \leq x \leq 4.06, x \in \mathrm{R}\}$, range: $\{y \mid 0 \leq y \leq 68, y \in \mathrm{R}\}$

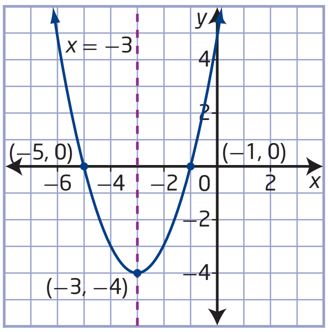
c) Example: The domain and range of algebraic functions may include all real values. For given real-world situations, the domain and range are determined by physical constraints such as time must be greater than or equal to zero and the height must be above ground, or greater than or equal to zero.

10. Examples:

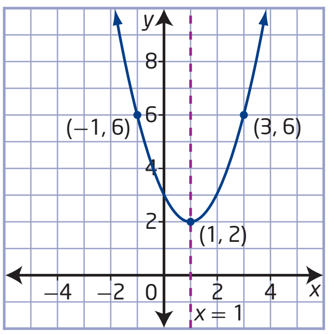
a)



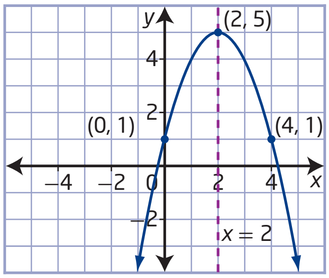
b)



c)

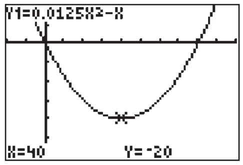


d)



11. a) $\{x \mid 0 \leq x \leq 80, x \in \mathrm{R}\}$

b)

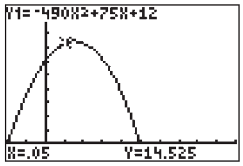


c) The maximum depth of the dish is 20 cm, which is the y-coordinate of the vertex $(40, -20)$. This is not the maximum value of the function. Since the parabola opens upward, this the minimum value of the function.

d) $\{d \mid -20 \leq d \leq 0, d \in \mathrm{R}\}$

e) The depth is approximately 17.19 cm, 25 cm from the edge of the dish.

12. a)



b) The h-intercept represents the height of the log.

c) 0.1 s; 14.9 cm

d) 0.3 s

e) domain: $\{t \mid 0 \leq t \leq 0.3, t \in \mathrm{R}\}$, range: $\{h \mid 0 \leq h \leq 14.9, h \in \mathrm{R}\}$

f) 14.5 cm

13. Examples:

a) $\{v \mid 0 \leq v \leq 150, v \in \mathrm{R}\}$

b)

v	f
0	0
25	1.25
50	5
75	11.25
100	20
125	31.25
150	45