Key Ideas

- You can solve some quadratic equations by factoring.
- If two factors of a quadratic equation have a product of zero, then by the zero product property one of the factors must be equal to zero.
- To solve a quadratic equation by factoring, first write the equation in the form $ax^2 + bx + c = 0$, and then factor the left side. Next, set each factor equal to zero, and solve for the unknown.

  For example,

  $$x^2 + 8x = -12$$
  $$x^2 + 8x + 12 = 0$$
  $$(x + 2)(x + 6) = 0$$
  $$x + 2 = 0 \quad \text{or} \quad x + 6 = 0$$
  $$x = -2 \qquad\qquad x = -6$$

- The solutions to a quadratic equation are called the roots of the equation.
- You can factor polynomials in quadratic form.
  - Factor trinomials of the form $a(P)^2 + b(P) + c$, where $a \neq 0$ and P is any expression, by replacing the expression for P with a single variable. Then substitute the expression for P back into the factored expression. Simplify the final factors, if possible.

    For example, factor $2(x + 3)^2 - 11(x + 3) + 15$ by letting $r = x + 3$.

    $$\begin{aligned} 2(x + 3)^2 - 11(x + 3) + 15 &= 2r^2 - 11r + 15 \\ &= 2r^2 - 5r - 6r + 15 \\ &= (2r^2 - 5r) + (-6r + 15) \\ &= r(2r - 5) - 3(2r - 5) \\ &= (2r - 5)(r - 3) \\ &= [2(x + 3) - 5][(x + 3) - 3] \\ &= (2x + 1)(x) \\ &= x(2x + 1) \end{aligned}$$

  - Factor a difference of squares, $P^2 - Q^2$, where P and Q are any expressions, as $[P - Q][P + Q]$.

Check Your Understanding

Practise

1. Factor completely.

a) $x^2 + 7x + 10$

b) $5z^2 + 40z + 60$

c) $0.2d^2 - 2.2d + 5.6$

2. Factor completely.

a) $3y^2 + 4y - 7$

b) $8k^2 - 6k - 5$

c) $0.4m^2 + 0.6m - 1.8$

3. Factor completely.

a) $x^2 + x - 20$

b) $x^2 - 12x + 36$

c) $\frac{1}{4}x^2 + 2x + 3$

d) $2x^2 + 12x + 18$

4. Factor each expression.

a) $4y^2 - 9x^2$

b) $0.36p^2 - 0.49q^2$

c) $\frac{1}{4}s^2 - \frac{9}{25}t^2$

d) $0.16t^2 - 16s^2$

5. Factor each expression.

a) $(x + 2)^2 - (x + 2) - 42$

b) $6(x^2 - 4x + 4)^2 + (x^2 - 4x + 4) - 1$

c) $(4j - 2)^2 - (2 + 4j)^2$

6. What are the factors of each expression?

a) $4(5b - 3)^2 + 10(5b - 3) - 6$

b) $16(x^2 + 1)^2 - 4(2x)^2$

c) $-\frac{1}{4}(2x)^2 + 25(2y^3)^2$

7. Solve each factored equation.

a) $(x + 3)(x + 4) = 0$

b) $(x - 2)\left(x + \frac{1}{2}\right) = 0$

c) $(x + 7)(x - 8) = 0$

d) $x(x + 5) = 0$

e) $(3x + 1)(5x - 4) = 0$

f) $2(x - 4)(7 - 2x) = 0$

8. Solve each quadratic equation by factoring. Check your answers.

a) $10n^2 - 40 = 0$

b) $\frac{1}{4}x^2 + \frac{5}{4}x + 1 = 0$

c) $3w^2 + 28w + 9 = 0$

d) $8y^2 - 22y + 15 = 0$

e) $d^2 + \frac{5}{2}d + \frac{3}{2} = 0$

f) $4x^2 - 12x + 9 = 0$

9. Determine the roots of each quadratic equation. Verify your answers.

a) $k^2 - 5k = 0$

b) $9x^2 = x + 8$

c) $\frac{8}{3}t + 5 = -\frac{1}{3}t^2$

d) $\frac{25}{49}y^2 - 9 = 0$

e) $2s^2 - 4s = 70$

f) $4q^2 - 28q = -49$

10. Solve each equation.

a) $42 = x^2 - x$

b) $g^2 = 30 - 7g$

c) $y^2 + 4y = 21$

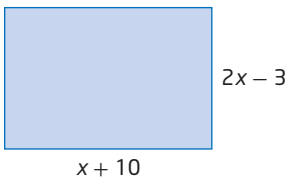
d) $3 = 6p^2 - 7p$

e) $3x^2 + 9x = 30$

f) $2z^2 = 3 - 5z$

Apply

11. A rectangle has dimensions $x + 10$ and $2x - 3$, where x is in centimetres. The area of the rectangle is 54 cm^2.



a) What equation could you use to determine the value of x?

b) What is the value of x?

12. An osprey, a fish-eating bird of prey, dives toward the water to catch a salmon. The height, h, in metres, of the osprey above the water t seconds after it begins its dive can be approximated by the function $h(t) = 5t^2 - 30t + 45$.

a) Determine the time it takes for the osprey to reach a height of 20 m.

b) What assumptions did you make? Are your assumptions reasonable? Explain.

13. A flare is launched from a boat. The height, h, in metres, of the flare above the water is approximately modelled by the function $h(t) = 150t - 5t^2$, where t is the number of seconds after the flare is launched.

a) What equation could you use to determine the time it takes for the flare to return to the water?

b) How many seconds will it take for the flare to return to the water?

14. The product of two consecutive even integers is 16 more than 8 times the smaller integer. Determine the integers.

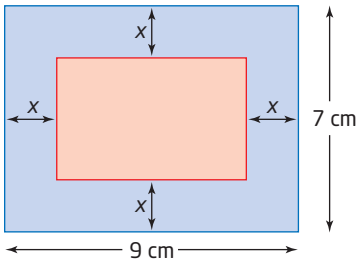
15. The area of a square is tripled by adding 10 cm to one dimension and 12 cm to the other. Determine the side length of the square.

16. Ted popped a baseball straight up with an initial upward velocity of 48 ft/s. The height, h, in feet, of the ball above the ground is modelled by the function $h(t) = 3 + 48t - 16t^2$. How long was the ball in the air if the catcher catches the ball 3 ft above the ground? Is your answer reasonable in this situation? Explain.

Did You Know?

Many Canadians have made a positive impact on Major League Baseball. Players such as Larry Walker of Maple Ridge, British Columbia, Jason Bay of Trail, British Columbia, and Justin Morneau of Westminster, British Columbia have had very successful careers in baseball's highest league.

17. A rectangle with area of 35 cm^2 is formed by cutting off strips of equal width from a rectangular piece of paper.



a) What is the width of each strip?

b) What are the dimensions of the new rectangle?

13. a) $k = 9$ **b)** $k < 9$ **c)** $k > 9$

14. a) 64 ft

b) The relationship between the height, radius, and span of the arch stays the same. Input the measures in metres and solve.

15. about 2.4 s

16. For the value of the function to change from negative to positive, it must cross the x-axis and therefore there must be an x-intercept between the two values of x.

17. The other x-intercept would have to be 4.

18. The x-coordinate of the vertex is halfway between the two roots. So, it is at 2. You can then substitute $x = 2$ into the equation to find the minimum value of -16.

4.2 Factoring Quadratic Equations, pages 229 to 233

1. a) $(x + 2)(x + 5)$ **b)** $5(z + 2)(z + 6)$

c) $0.2(d - 4)(d - 7)$

2. a) $(y - 1)(3y + 7)$ **b)** $(4k - 5)(2k + 1)$

c) $0.2(2m - 3)(m + 3)$

3. a) $(x + 5)(x - 4)$ **b)** $(x - 6)^2$

c) $\frac{1}{4}(x + 2)(x + 6)$ **d)** $2(x + 3)^2$

4. a) $(2y + 3x)(2y - 3x)$

b) $(0.6p + 0.7q)(0.6p - 0.7q)$

c) $\left(\frac{1}{2}s + \frac{3}{5}t\right)\left(\frac{1}{2}s - \frac{3}{5}t\right)$

d) $(0.4t + 4s)(0.4t - 4s)$

5. a) $(x + 8)(x - 5)$

b) $(2x^2 - 8x + 9)(3x^2 - 12x + 11)$

c) $(-4)(8j)$

6. a) $(10b)(10b - 7)$

b) $16(x^2 - x + 1)(x^2 + x + 1)$

c) $(10y^3 - x)(10y^3 + x)$

7. a) $x = -3, x = -4$ **b)** $x = 2, x = -\frac{1}{2}$

c) $x = -7, x = 8$ **d)** $x = 0, x = -5$

e) $x = -\frac{1}{3}, x = \frac{4}{5}$ **f)** $x = 4, x = \frac{7}{2}$

8. a) $n = -2, n = 2$ **b)** $x = -4, x = -1$

c) $w = -9, x = -\frac{1}{3}$ **d)** $y = \frac{5}{4}, y = \frac{3}{2}$

e) $d = -\frac{3}{2}, d = -1$ **f)** $x = \frac{3}{2}$

9. a) 0 and 5 **b)** $-\frac{8}{9}$ and 1

c) -5 and -3 **d)** $-\frac{21}{5}$ and $\frac{21}{5}$

e) -5 and 7 **f)** $\frac{7}{2}$

10. a) -6 and 7 **b)** -10 and 3

c) -7 and 3 **d)** $-\frac{1}{3}$ and $\frac{3}{2}$

e) -5 and 2 **f)** -3 and $\frac{1}{2}$

11. a) $(x + 10)(2x - 3) = 54$ **b)** 3.5 cm

12. a) 1 s and 5 s

b) Assume that the mass of the fish does not affect the speed at which the osprey flies after catching the fish. This may not be a reasonable assumption for a large fish.

13. a) $150t - 5t^2 = 0$ **b)** 30 s

14. 8 and 10 or 0 and -2

15. 15 cm

16. 3 s; this seems a very long time considering the ball went up only 39 ft.

17. a) 1 cm

b) 7 cm by 5 cm

18. a) No; $(x - 5)$ is not a factor of the expression $x^2 - 5x - 36$, since $x = 5$ does not satisfy the equation $x^2 - 5x - 36 = 0$.

b) Yes; $(x + 3)$ is a factor of the expression $x^2 - 2x - 15$, since $x = -3$ satisfies the equation $x^2 - 2x - 15 = 0$.

c) No; $(4x + 1)$ is not a factor of the expression $6x^2 + 11x + 4$, since $x = -\frac{1}{4}$ does not satisfy the equation $6x^2 + 11x + 4 = 0$.

d) Yes; $(2x - 1)$ is a factor of the expression $4x^2 + 4x - 3$, since $x = \frac{1}{2}$ satisfies the equation $4x^2 + 4x - 3 = 0$.

19. a) $-\frac{1}{2}$ and 2 **b)** -4 and 3

20. 20 cm and 21 cm

21. 8 m and 15 m

22. a) $x(x - 7) = 690$ **b)** 30 cm by 23 cm

23. 5 m

24. 5 m

25. $P = \frac{1}{2}d(v_1 + v_2)(v_1 - v_2)$

26. No; the factor $6x - 4$ still has a common factor of 2.

27. a) $6(z - 1)(2z + 5)$

b) $4(2m^2 - 8 - 3n)(2m^2 - 8 + 3n)$

c) $\frac{1}{36}(2y - 3x)^2$

d) $7\left(w - \frac{5}{3}\right)(5w + 1)$

28. $4(3x + 5y)$ centimetres

29. The shop will make a profit after 4 years.

30. a) $x^2 - 9 = 0$ **b)** $x^2 - 4x + 4 = 0$

c) $3x^2 - 14x + 8 = 0$

d) $10x^2 - x - 3 = 0$

31. Example: $x^2 - x + 1 = 0$

32. a) Instead of evaluating $81 - 36$, use the difference of squares pattern to rewrite the expression as $(9 - 6)(9 + 6)$ and then simplify. You can use this method when a question asks you to subtract a square number from a square number.