Key Ideas

- You can compare and order radicals using a variety of strategies:
  - Convert unlike radicals to entire radicals. If the radicals have the same index, the radicands can be compared.
  - Compare the coefficients of like radicals.
  - Compare the indices of radicals with equal radicands.
- When adding or subtracting radicals, combine coefficients of like radicals. In general, $m\sqrt[r]{a} + n\sqrt[r]{a} = (m + n)\sqrt[r]{a}$, where r is a natural number, and m, n, and a are real numbers. If r is even, then $a \geq 0$.
- A radical is in simplest form if the radicand does not contain a fraction or any factor which may be removed, and the radical is not part of the denominator of a fraction.

  For example, $$\begin{aligned} 5\sqrt{40} &= 5\sqrt{4(10)} \\ &= 5\sqrt{4}\left(\sqrt{10}\right) \\ &= 5(2)\sqrt{10} \\ &= 10\sqrt{10} \end{aligned}$$
- When a radicand contains variables, identify the values of the variables that make the radical a real number by considering the index and the radicand:
  - If the index is an even number, the radicand must be non-negative.

    For example, in $\sqrt{3n}$, the index is even. So, the radicand must be non-negative.
    $3n \geq 0$
    $n \geq 0$
  - If the index is an odd number, the radicand may be any real number.

    For example, in $\sqrt[3]{x}$, the index is odd. So, the radicand, x, can be any real number—positive, negative, or zero.

Check Your Understanding

Practise

1. Copy and complete the table.

Mixed Radical Form	Entire Radical Form
$4\sqrt{7}$	
	$\sqrt{50}$
$-11\sqrt{8}$	
	$-\sqrt{200}$

2. Express each radical as a mixed radical in simplest form.
   a) $\sqrt{56}$ b) $3\sqrt{75}$
   c) $\sqrt[3]{24}$ d) $\sqrt{c^3d^2}$, $c \geq 0$, $d \geq 0$
3. Write each expression in simplest form. Identify the values of the variable for which the radical represents a real number.
   a) $3\sqrt{8m^4}$ b) $\sqrt[3]{24q^5}$
   c) $-2\sqrt[5]{160s^5t^6}$

4. Copy and complete the table. State the values of the variable for which the radical represents a real number.

Mixed Radical Form	Entire Radical Form
$3n\sqrt{5}$	
	$\sqrt[3]{-432}$
$\frac{1}{2a}\sqrt[3]{7a}$	
	$\sqrt[3]{128x^4}$

5. Express each pair of terms as like radicals. Explain your strategy.

a) $15\sqrt{5}$ and $8\sqrt{125}$

b) $8\sqrt{112z^8}$ and $48\sqrt{7z^4}$

c) $-35\sqrt[4]{w^2}$ and $3\sqrt[4]{81w^{10}}$

d) $6\sqrt[3]{2}$ and $6\sqrt[3]{54}$

6. Order each set of numbers from least to greatest.

a) $3\sqrt{6}$, 10, and $7\sqrt{2}$

b) $-2\sqrt{3}$, -4, $-3\sqrt{2}$, and $-2\sqrt{\frac{7}{2}}$

c) $\sqrt[3]{21}$, $3\sqrt[3]{2}$, 2.8, $2\sqrt[3]{5}$

7. Verify your answer to #6b) using a different method.

8. Simplify each expression.

a) $-\sqrt{5} + 9\sqrt{5} - 4\sqrt{5}$

b) $1.4\sqrt{2} + 9\sqrt{2} - 7$

c) $\sqrt[4]{11} - 1 - 5\sqrt[4]{11} + 15$

d) $-\sqrt{6} + \frac{9}{2}\sqrt{10} - \frac{5}{2}\sqrt{10} + \frac{1}{3}\sqrt{6}$

9. Simplify.

a) $3\sqrt{75} - \sqrt{27}$

b) $2\sqrt{18} + 9\sqrt{7} - \sqrt{63}$

c) $-8\sqrt{45} + 5.1 - \sqrt{80} + 17.4$

d) $\frac{2}{3}\sqrt[3]{81} + \frac{\sqrt[3]{375}}{4} - 4\sqrt{99} + 5\sqrt{11}$

10. Simplify each expression. Identify any restrictions on the values for the variables.

a) $2\sqrt{a^3} + 6\sqrt{a^3}$

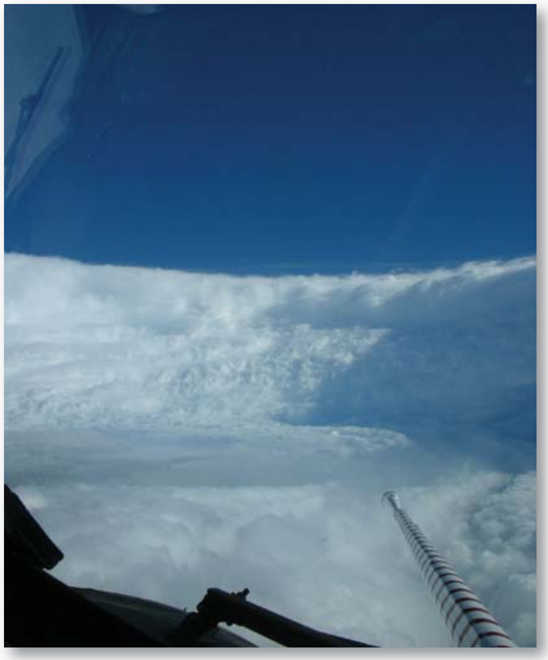
b) $3\sqrt{2x} + 3\sqrt{8x} - \sqrt{x}$

c) $-4\sqrt[3]{625r} + \sqrt[3]{40r^4}$

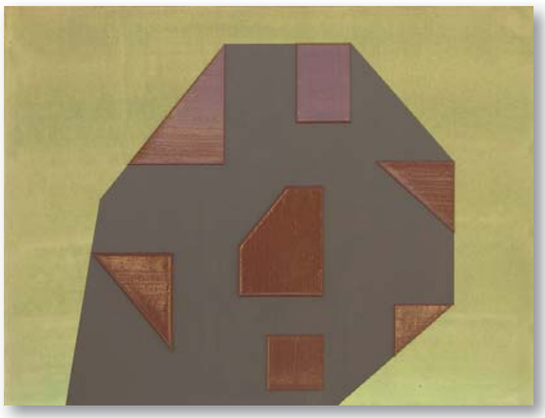
d) $\frac{w}{5}\sqrt[3]{-64} + \frac{\sqrt[3]{512w^3}}{5} - \frac{2}{5}\sqrt{50w} - 4\sqrt{2w}$

Apply

11. The air pressure, p, in millibars (mbar) at the centre of a hurricane, and wind speed, w, in metres per second, of the hurricane are related by the formula $w = 6.3\sqrt{1013 - p}$. What is the exact wind speed of a hurricane if the air pressure is 965 mbar?



12. Saskatoon artist Jonathan Forrest's painting, *Clincher*, contains geometric shapes. The isosceles right triangle at the top right has legs that measure approximately 12 cm. What is the length of the hypotenuse? Express your answer as a radical in simplest form.



Clincher, by Jonathan Forrest Saskatoon, Saskatchewan

d) $y = \frac{1}{4}(x + 8)^2 + 4$; the shape of the graph of $y = \frac{1}{4}(x + 8)^2 + 4$ is wider by a multiplication of the y-values by a factor of $\frac{1}{4}$ and translated 8 units to the left and 4 units up.

6. a) 22 m b) 2 m c) 4 s
7. In order: roots, zeros, x-intercepts
8. a) $(3x + 4)(3x - 2)$ b) $(4r - 9s)(4r + 9s)$
   c) $(x + 3)(2x + 9)$ d) $(xy + 4)(xy - 9)$
   e) $5(a + b)(13a + b)$ f) $(11r + 20)(11r - 20)$
9. 7, 8, 9 or -9, -8, -7
10. 15 seats per row, 19 rows
11. 3.5 m
12. Example: Dallas did not divide the 2 out of the -12 in the first line or multiply the 36 by 2 and thus add 72 to the right side instead of 36 in line two. Doug made a sign error on the -12 in the first line. He should have calculated 200 as the value in the radical, not 80. When he simplified, he took $\sqrt{80}$ divided by 4 to get $\sqrt{20}$, which is not correct.

    The correct answer is $3 \pm \frac{5}{\sqrt{2}}$ or $\frac{6 \pm 5\sqrt{2}}{2}$.
13. a) Example: square root, $x = \pm\sqrt{2}$
    b) Example: factor, $m = 2$ and $m = 13$
    c) Example: factor, $s = -5$ and $s = 7$
    d) Example: use quadratic formula, $x = -\frac{1}{16}$ and $x = 3$
14. a) two distinct real roots
    b) one distinct real root
    c) no real roots
15. a) $85 = x^2 + (x + 1)^2$
    b) Example: factoring, $x = -7$ and $x = 6$
    c) The top is 7-in. by 7-in. and the bottom is 6-in. by 6-in.
    d) Example: Negative lengths are not possible.

Unit 2 Test, pages 266 to 267

1. A
2. D
3. D
4. B
5. B
6. 76
7. $900
8. 0.18
9. a) 53.5 cm b) 75.7 cm c) No
10. a) 47.5 m b) 6.1 s
11. 12 cm by 12 cm
12. a) $3x^2 + 6x - 672 = 0$
    b) $x = -16$ and $x = 14$
    c) 14 in., 15 in., and 16 in.
    d) Negative lengths are not possible.

Chapter 5 Radical Expressions and Equations

5.1 Working With Radicals, pages 278 to 281

1.

Mixed Radical Form	Entire Radical Form
$4\sqrt{7}$	$\sqrt{112}$
$5\sqrt{2}$	$\sqrt{50}$
$-11\sqrt{8}$	$-\sqrt{968}$
$-10\sqrt{2}$	$-\sqrt{200}$

2. a) $2\sqrt{14}$ b) $15\sqrt{3}$
   c) $2\sqrt[3]{3}$ d) $cd\sqrt{c}$
3. a) $6m^2\sqrt{2}$, $m \in R$ b) $2q\sqrt[3]{3q^2}$, $q \in R$
   c) $-4st\sqrt[5]{5t}$, $s, t \in R$
4.

Mixed Radical Form	Entire Radical Form
$3n\sqrt{5}$	$\sqrt{45n^2}, n \geq 0$ or $-\sqrt{45n^2}, n < 0$
$-6\sqrt[3]{2}$	$\sqrt[3]{-432}$
$\frac{1}{2a}\sqrt[3]{7a}$	$\sqrt[3]{\frac{7}{8a^2}}, a \neq 0$
$4x\sqrt[3]{2x}$	$\sqrt[3]{128x^4}$

5. a) $15\sqrt{5}$ and $40\sqrt{5}$ b) $32z^4\sqrt{7}$ and $48z^2\sqrt{7}$
   c) $-35\sqrt[4]{w^2}$ and $9w^2(\sqrt[4]{w^2})$
   d) $6\sqrt[3]{2}$ and $18\sqrt[3]{2}$
6. a) $3\sqrt{6}$, $7\sqrt{2}$, 10
   b) $-3\sqrt{2}$, -4, $-2\sqrt{\frac{7}{2}}$, $-2\sqrt{3}$
   c) $\sqrt[3]{21}$, 2.8, $2\sqrt[3]{5}$, $3\sqrt[3]{2}$
7. Example: Technology could be used.
8. a) $4\sqrt{5}$ b) $10.4\sqrt{2} - 7$
   c) $-4\sqrt[4]{11} + 14$ d) $-\frac{2}{3}\sqrt{6} + 2\sqrt{10}$
9. a) $12\sqrt{3}$ b) $6\sqrt{2} + 6\sqrt{7}$
   c) $-28\sqrt{5} + 22.5$ d) $\frac{13}{4}\sqrt[3]{3} - 7\sqrt{11}$
10. a) $8a\sqrt{a}$, $a \geq 0$ b) $9\sqrt{2x} - \sqrt{x}$, $x \geq 0$
    c) $2(r - 10)\sqrt[3]{5r}$, $r \in R$
    d) $\frac{4w}{5} - 6\sqrt{2w}$, $w \geq 0$
11. $25.2\sqrt{3}$ m/s
12. $12\sqrt{2}$ cm
13. $12\sqrt[3]{3025}$ million kilometres
14. $2\sqrt{30}$ m/s $\approx$ 11 m/s
15. a) $2\sqrt{38}$ m b) $8\sqrt{19}$ m
16. $\sqrt{1575}$ mm^2, $15\sqrt{7}$ mm^2
17. $7\sqrt{5}$ units
18. $14\sqrt{2}$ m
19. Brady is correct. The answer can be further simplified to $10y^2\sqrt{y}$.
20. $4\sqrt{58}$

    Example: Simplify each radical to see which is not a like radical to $12\sqrt{6}$.
21. $\sqrt{2} - \sqrt{3}$ m