Key Ideas

- When multiplying radicals with identical indices, multiply the coefficients and multiply the radicands:
  $$(m\sqrt[k]{a})(n\sqrt[k]{b}) = mn\sqrt[k]{ab}$$
  where k is a natural number, and m, n, a, and b are real numbers. If k is even, then $a \geq 0$ and $b \geq 0$.
- When dividing two radicals with identical indices, divide the coefficients and divide the radicands:
  $$\frac{m\sqrt[k]{a}}{n\sqrt[k]{b}} = \frac{m}{n}\sqrt[k]{\frac{a}{b}}$$
  where k is a natural number, and m, n, a, and b are real numbers. $n \neq 0$ and $b \neq 0$. If k is even, then $a \geq 0$ and b > 0.
- When multiplying radical expressions with more than one term, use the distributive property and then simplify.
- To rationalize a monomial denominator, multiply the numerator and denominator by an expression that produces a rational number in the denominator.
  $$\frac{2}{\sqrt[5]{n}}\left(\frac{(\sqrt[5]{n})^4}{(\sqrt[5]{n})^4}\right) = \frac{2(\sqrt[5]{n})^4}{n}$$
- To simplify an expression with a square-root binomial in the denominator, rationalize the denominator using these steps:
  - Determine a conjugate of the denominator.
  - Multiply the numerator and denominator by this conjugate.
  - Express in simplest form.

Check Your Understanding

Practise

1. Multiply. Express all products in simplest form.

a) $2\sqrt{5}(7\sqrt{3})$

b) $-\sqrt{32}(7\sqrt{2})$

c) $2\sqrt[4]{48}(\sqrt[4]{5})$

d) $4\sqrt{19x}(\sqrt{2x^2})$, $x \geq 0$

e) $\sqrt[3]{54y^7}(\sqrt[3]{6y^4})$

f) $\sqrt{6t}\left(3t^2\sqrt{\frac{t}{4}}\right)$, $t \geq 0$

2. Multiply using the distributive property. Then, simplify.

a) $\sqrt{11}(3 - 4\sqrt{7})$

b) $-\sqrt{2}(14\sqrt{5} + 3\sqrt{6} - \sqrt{13})$

c) $\sqrt{y}(2\sqrt{y} + 1)$, $y \geq 0$

d) $z\sqrt{3}(z\sqrt{12} - 5z + 2)$

3. Simplify. Identify the values of the variables for which the radicals represent real numbers.

a) $-3(\sqrt{2} - 4) + 9\sqrt{2}$

b) $7(-1 - 2\sqrt{6}) + 5\sqrt{6} + 8$

c) $4\sqrt{5}(\sqrt{3j} + 8) - 3\sqrt{15j} + \sqrt{5}$

d) $3 - \sqrt[3]{4k}(12 + 2\sqrt[3]{8})$

4. Expand and simplify each expression.

a) $(8\sqrt{7} + 2)(\sqrt{2} - 3)$

b) $(4 - 9\sqrt{5})(4 + 9\sqrt{5})$

c) $(\sqrt{3} + 2\sqrt{15})(\sqrt{3} - \sqrt{15})$

d) $(6\sqrt[3]{2} - 4\sqrt{13})^2$

e) $(-\sqrt{6} + 2)(2\sqrt{2} - 3\sqrt{5} + 1)$

5. Expand and simplify. State any restrictions on the values for the variables.

a) $(15\sqrt{c} + 2)(\sqrt{2c} - 6)$

b) $(1 - 10\sqrt{8x^3})(2 + 7\sqrt{5x})$

c) $(9\sqrt{2m} - 4\sqrt{6m})^2$

d) $(10r - 4\sqrt[3]{4r})(2\sqrt[3]{6r^2} + 3\sqrt[3]{12r})$

6. Divide. Express your answers in simplest form.

a) $\frac{\sqrt{80}}{\sqrt{10}}$

b) $\frac{-2\sqrt{12}}{4\sqrt{3}}$

c) $\frac{3\sqrt{22}}{\sqrt{11}}$

d) $\frac{3\sqrt{135m^5}}{\sqrt{21m^3}}$, $m > 0$

7. Simplify.

a) $\frac{9\sqrt{432p^5} - 7\sqrt{27p^5}}{\sqrt{33p^4}}$, $p > 0$

b) $\frac{6\sqrt[3]{4v^7}}{\sqrt[3]{14v}}$, $v > 0$

8. Rationalize each denominator. Express each radical in simplest form.

a) $\frac{20}{\sqrt{10}}$

b) $\frac{-\sqrt{21}}{\sqrt{7m}}$, $m > 0$

c) $-\frac{2}{3}\sqrt{\frac{5}{12u}}$, $u > 0$

d) $20\sqrt[3]{\frac{6t}{5}}$

9. Determine a conjugate for each binomial. What is the product of each pair of conjugates?

a) $2\sqrt{3} + 1$

b) $7 - \sqrt{11}$

c) $8\sqrt{z} - 3\sqrt{7}$, $z \geq 0$

d) $19\sqrt{h} + 4\sqrt{2h}$, $h \geq 0$

10. Rationalize each denominator. Simplify.

a) $\frac{5}{2 - \sqrt{3}}$

b) $\frac{7\sqrt{2}}{\sqrt{6} + 8}$

c) $\frac{-\sqrt{7}}{\sqrt{5} - 2\sqrt{2}}$

d) $\frac{\sqrt{3} + \sqrt{13}}{\sqrt{3} - \sqrt{13}}$

11. Write each fraction in simplest form. Identify the values of the variables for which each fraction is a real number.

a) $\frac{4r}{\sqrt{6r} + 9}$

b) $\frac{18\sqrt{3n}}{\sqrt{24n}}$

c) $\frac{8}{4 - \sqrt{6t}}$

d) $\frac{5\sqrt{3y}}{\sqrt{10} + 2}$

12. Use the distributive property to simplify $(c + c\sqrt{c})(c + 7\sqrt{3c})$, $c \geq 0$.

Apply

13. Malcolm tries to rationalize the denominator in the expression $\frac{4}{3 - 2\sqrt{2}}$ as shown below.

a) Identify, explain, and correct any errors.

b) Verify your corrected solution.

Malcolm's solution:

$$\frac{4}{3 - 2\sqrt{2}} = \left(\frac{4}{3 - 2\sqrt{2}}\right)\left(\frac{3 + 2\sqrt{2}}{3 + 2\sqrt{2}}\right)$$
$$= \frac{12 + 8\sqrt{4(2)}}{9 - 8}$$
$$= 12 + 16\sqrt{2}$$

22. $12\sqrt{2}$ cm

23. $5\sqrt{3}$ and $7\sqrt{3}$
It is an arithmetic sequence with a common difference of $2\sqrt{3}$.

24. a) $2\sqrt{75}$ and $108^{\frac{1}{2}}$ Example: Write the radicals in simplest form; then, add the two radicals with the greatest coefficients.
b) $2\sqrt{75}$ and $-3\sqrt{12}$ Example: Write the radicals in simplest form; then, subtract the radical with the least coefficient from the radical with the greatest coefficient.

25. a) Example: If $x = 3$,
$(-3)^2 = (-3)(-3)$
$(-3)^2 = 9$
$(-3)^2 = 3^2$
b) Example: If $x = 3$,
$\sqrt{3^2} = \sqrt{9}$
$\sqrt{9} = 3$
$\sqrt{3^2} \neq -3$

5.2 Multiplying and Dividing Radical Expressions, pages 289 to 293

1. a) $14\sqrt{15}$ **b)** -56 **c)** $4\sqrt[4]{15}$
d) $4x\sqrt{38x}$ **e)** $3y^3\left(\sqrt[3]{12y^2}\right)$ **f)** $\frac{3t^3}{2}\sqrt{6}$

2. a) $3\sqrt{11} - 4\sqrt{77}$
b) $-14\sqrt{10} - 6\sqrt{3} + \sqrt{26}$
c) $2y + \sqrt{y}$ **d)** $6z^2 - 5z^2\sqrt{3} + 2z\sqrt{3}$

3. a) $6\sqrt{2} + 12$ **b)** $1 - 9\sqrt{6}$
c) $\sqrt{15j} + 33\sqrt{5}$, $j \geq 0$ **d)** $3 - 16\sqrt[3]{4k}$

4. a) $8\sqrt{14} - 24\sqrt{7} + 2\sqrt{2} - 6$
b) -389
c) $-27 + 3\sqrt{5}$
d) $36\sqrt[3]{4} - 48\sqrt{13}(\sqrt[3]{2}) + 208$
e) $-4\sqrt{3} + 3\sqrt{30} - \sqrt{6} + 4\sqrt{2} - 6\sqrt{5} + 2$

5. a) $15c\sqrt{2} - 90\sqrt{c} + 2\sqrt{2c} - 12$, $c \geq 0$
b) $2 + 7\sqrt{5x} - 40x\sqrt{2x} - 140x^2\sqrt{10}$, $x \geq 0$
c) $258m - 144m\sqrt{3}$, $m \geq 0$
d) $20r\sqrt[3]{6r^2} + 30r\sqrt[3]{12r} - 16r\sqrt[3]{3} - 24\sqrt[3]{6r^2}$

6. a) $2\sqrt{2}$ **b)** -1
c) $3\sqrt{2}$ **d)** $\frac{9m\sqrt{35}}{7}$

7. a) $\frac{87\sqrt{11p}}{11}$ **b)** $\frac{6v^2\sqrt[3]{98}}{7}$

8. a) $2\sqrt{10}$ **b)** $\frac{-\sqrt{3m}}{m}$
c) $\frac{-\sqrt{15u}}{9u}$ **d)** $4\sqrt[3]{150t}$

9. a) $2\sqrt{3} - 1$; 11 **b)** $7 + \sqrt{11}$; 38
c) $8\sqrt{z} + 3\sqrt{7}$; $64z - 63$
d) $19\sqrt{h} - 4\sqrt{2h}$; $329h$

10. a) $10 + 5\sqrt{3}$ **b)** $\frac{-7\sqrt{3} + 28\sqrt{2}}{29}$
c) $\frac{\sqrt{35} + 2\sqrt{14}}{3}$ **d)** $\frac{-8 - \sqrt{39}}{5}$

11. a) $\frac{4r^2\sqrt{6} - 36r}{6r^2 - 81}$, $r \neq \frac{\pm 3\sqrt{6}}{2}$
b) $\frac{9\sqrt{2}}{2}$, $n > 0$
c) $\frac{16 + 4\sqrt{6t}}{8 - 3t}$, $t \neq \frac{8}{3}$, $t \geq 0$
d) $\frac{5\sqrt{30y} - 10\sqrt{3y}}{6}$, $y \geq 0$

12. $c^2 + 7c\sqrt{3c} + c^2\sqrt{c} + 7c^2\sqrt{3}$

13. a) When applying the distributive property, Malcolm distributed the 4 to both the whole number and the root. The 4 should only be distributed to the whole number. The correct answer is $12 + 8\sqrt{2}$.
b) Example: Verify using decimal approximations.
$\frac{4}{3 - 2\sqrt{2}} \approx 23.3137$
$12 + 8\sqrt{2} \approx 23.3137$

14. $\frac{\sqrt{5} + 1}{2}$

15. a) $T = \frac{\pi\sqrt{10L}}{5}$ **b)** $\frac{9\pi\sqrt{30}}{5}$ s

16. $860 + 172\sqrt{5}$ m

17. $-28 - 16\sqrt{3}$

18. a) $4\sqrt[3]{3}$ mm **b)** $2\sqrt[3]{6}$ mm **c)** $2\sqrt[3]{3} : \sqrt[3]{6}$

19. a) Lev forgot to switch the inequality sign when he divided by -5. The correct answer is $x < \frac{3}{5}$.
b) The square root of a negative number is not a real number.
c) Example: The expression cannot have a variable in the denominator or under the radical sign. $\frac{2x\sqrt{14}}{3\sqrt{5}}$

20. Olivia evaluated $\sqrt{25}$ as ± 5 in the third step. The final steps should be as follows:
$$\frac{\sqrt{3}(2c - 5c)}{3} = \frac{\sqrt{3}(-3c)}{3}$$
$$= -c\sqrt{3}$$

21. 735 cm^3

22. 12 m^2

23. $\left(\frac{15\sqrt{3}}{2}, \frac{9\sqrt{2}}{2}\right)$

24. $\frac{25x^2 + 30x\sqrt{x} + 9x}{625x^2 - 450x + 81}$ or $\frac{x(25x + 30\sqrt{x} + 9)}{(25x - 9)^2}$

25. a) $-3 \pm \sqrt{6}$ **b)** -6 **c)** 3
d) Examples: The answer to part b) is the opposite value of the coefficient of the middle term. The answer to part c) is the value of the constant.

26. $\frac{(\sqrt[c]{a})(\sqrt[n-1]{r})}{r}$

27. $(15\sqrt{14} + 42\sqrt{7} + 245\sqrt{2} + 7\sqrt{2702})$ cm^2

28. Example: You cannot multiply or divide radical expressions with different indices, or algebraic expressions with different variables.