Key Ideas

- You can model some real-world relationships with radical equations.
- When solving radical equations, begin by isolating one of the radical terms.
- To eliminate a square root, raise both sides of the equation to the exponent two. For example, in $3 = \sqrt{c + 5}$, square both sides.

  $3^2 = \left(\sqrt{c + 5}\right)^2$
  $9 = c + 5$
  $4 = c$

- To identify whether a root is extraneous, substitute the value into the original equation. Raising both sides of an equation to an even exponent may introduce an extraneous root.
- When determining restrictions on the values for variables, consider the following:
  - Denominators cannot be equal to zero.
  - For radicals to be real numbers, radicands must be non-negative if the index is an even number.

Check Your Understanding

Practise

Determine any restrictions on the values for the variable in each radical equation, unless given.

1. Square each expression.

a) $\sqrt{3z}$, $z \geq 0$

b) $\sqrt{x - 4}$, $x \geq 4$

c) $2\sqrt{x + 7}$, $x \geq -7$

d) $-4\sqrt{9 - 2y}$, $\frac{9}{2} \geq y$

2. Describe the steps to solve the equation $\sqrt{x} + 5 = 11$, where $x \geq 0$.

3. Solve each radical equation. Verify your solutions and identify any extraneous roots.

a) $\sqrt{2x} = 3$

b) $\sqrt{-8x} = 4$

c) $7 = \sqrt{5 - 2x}$

4. Solve each radical equation. Verify your solutions.

a) $\sqrt{z} + 8 = 13$

b) $2 - \sqrt{y} = -4$

c) $\sqrt{3x} - 8 = -6$

d) $-5 = 2 - \sqrt{-6m}$

5. In the solution to $k + 4 = \sqrt{-2k}$, identify whether either of the values, $k = -8$ or $k = -2$, is extraneous. Explain your reasoning.

6. Isolate each radical term. Then, solve the equation.

a) $-3\sqrt{n - 1} + 7 = -14$, $n \geq 1$

b) $-7 - 4\sqrt{2x - 1} = 17$, $x \geq \frac{1}{2}$

c) $12 = -3 + 5\sqrt{8 - x}$, $x \leq 8$

7. Solve each radical equation.

a) $\sqrt{m^2 - 3} = 5$

b) $\sqrt{x^2 + 12x} = 8$

c) $\sqrt{\frac{q^2}{2} + 11} = q - 1$

d) $2n + 2\sqrt{n^2 - 7} = 14$

8. Solve each radical equation.

a) $5 + \sqrt{3x - 5} = x$

b) $\sqrt{x^2 + 30x} = 8$

c) $\sqrt{d + 5} = d - 1$

d) $\sqrt{\frac{j + 1}{3}} + 5j = 3j - 1$

9. Solve each radical equation.

a) $\sqrt{2k} = \sqrt{8}$

b) $\sqrt{-3m} = \sqrt{-7m}$

c) $5\sqrt{\frac{j}{2}} = \sqrt{200}$

d) $5 + \sqrt{n} = \sqrt{3n}$

10. Solve.

a) $\sqrt{z + 5} = \sqrt{2z - 1}$

b) $\sqrt{6y - 1} = \sqrt{-17 + y^2}$

c) $\sqrt{5r - 9} - 3 = \sqrt{r + 4} - 2$

d) $\sqrt{x + 19} + \sqrt{x - 2} = 7$

Apply

11. By inspection, determine which one of the following equations will have an extraneous root. Explain your reasoning.

$\sqrt{3y - 1} - 2 = 5$

$4 - \sqrt{m + 6} = -9$

$\sqrt{x + 8} + 9 = 2$

12. The following steps show how Jerry solved the equation $3 + \sqrt{x + 17} = x$. Is his work correct? Explain your reasoning and provide a correct solution if necessary.

Jerry's Solution

$$3 + \sqrt{x + 17} = x$$
$$\sqrt{x + 17} = x - 3$$
$$\left(\sqrt{x + 17}\right)^2 = x^2 - 3^2$$
$$x + 17 = x^2 - 9$$
$$0 = x^2 - x - 26$$
$$x = \frac{1 \pm \sqrt{1 + 104}}{2}$$
$$x = \frac{1 \pm \sqrt{105}}{2}$$

13. Collision investigators can approximate the initial velocity, v, in kilometres per hour, of a car based on the length, l, in metres, of the skid mark. The formula $v = 12.6\sqrt{l} + 8$, $l \geq 0$, models the relationship. What length of skid is expected if a car is travelling 50 km/h when the brakes are applied? Express your answer to the nearest tenth of a metre.

14. In 1805, Rear-Admiral Beaufort created a numerical scale to help sailors quickly assess the strength of the wind. The integer scale ranges from 0 to 12. The wind scale, B, is related to the wind velocity, v, in kilometres per hour, by the formula $B = 1.33\sqrt{v + 10.0} - 3.49$, $v \geq -10$.

a) Determine the wind scale for a wind velocity of 40 km/h.

b) What wind velocity results in a wind scale of 3?

Web Link

To learn more about the Beaufort scale, go to www.mhrprecalc11.ca and follow the links.

29. Examples: To rationalize the denominator you need to multiply the numerator and denominator by a conjugate. To factor a difference of squares, each factor is the conjugate of the other. If you factor $3a - 16$ as a difference of squares, the factors are $\sqrt{3a} - 4$ and $\sqrt{3a} + 4$. The factors form a conjugate pair.

30. a) 3 m

b) $h(t) = -5(t - 1)^2 + 8$; $t = \sqrt{\dfrac{8 - h}{5}} + 1$

c) $\dfrac{19 + 4\sqrt{10}}{4}$ m

Example: The snowboarder starts the jump at $t = 0$ and ends the jump at $t = \dfrac{5 + 2\sqrt{10}}{5}$. The snowboarder will be halfway at $t = \dfrac{5 + 2\sqrt{10}}{10}$. Substitute this value of t into the original equation to find the height at the halfway point.

31. Yes, they are. Example: using the quadratic formula

32. a) $\dfrac{\sqrt[3]{6V(V - 1)^2}}{V - 1}$

b) A volume greater than one will result in a real ratio.

33. Step 1

$y = \sqrt{x}$

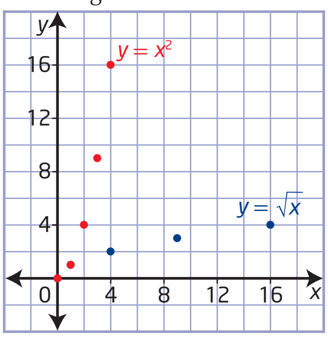
x	y
0	0
1	1
4	2
9	3
16	4

$y = x^2$

x	y
0	0
1	1
2	4
3	9
4	16

Step 2 Example: The values of x and y have been interchanged.

Step 3



Example: The restrictions on the radical function produce the right half of the parabola.

5.3 Radical Equations, pages 300 to 303

1. a) $3z$ **b)** $x - 4$ **c)** $4(x + 7)$ **d)** $16(9 - 2y)$

2. Example: Isolate the radical and square both sides. $x = 36$

3. a) $x = \dfrac{9}{2}$ **b)** $x = -2$ **c)** $x = -22$

4. a) $z = 25$ **b)** $y = 36$ **c)** $x = \dfrac{4}{3}$ **d)** $m = -\dfrac{49}{6}$

5. $k = -8$ is an extraneous root because if -8 is substituted for k, the result is a square root that equals a negative number, which cannot be true in the real-number system.

6. a) $n = 50$ **b)** no solution **c)** $x = -1$

7. a) $m = \pm 2\sqrt{7}$ **b)** $x = -16$, $x = 4$ **c)** $q = 2 + 2\sqrt{6}$ **d)** $n = 4$

8. a) $x = 10$ **b)** $x = -32$, $x = 2$ **c)** $d = 4$ **d)** $j = -\dfrac{2}{3}$

9. a) $k = 4$ **b)** $m = 0$ **c)** $j = 16$ **d)** $n = \dfrac{50 + 25\sqrt{3}}{2}$

10. a) $z = 6$ **b)** $y = 8$ **c)** $r = 5$ **d)** $x = 6$

11. The equation $\sqrt{x + 8} + 9 = 2$ has an extraneous root because simplifying it further to $\sqrt{x + 8} = -7$ has no solution.

12. Example: Jerry made a mistake when he squared both sides, because he squared each term on the right side rather than squaring $(x - 3)$. The right side should have been $(x - 3)^2 = x^2 - 6x + 9$, which gives $x = 8$ as the correct solution. Jerry should have listed the restriction following the first line: $x \geq -17$.

13. 11.1 m

14. a) $B \approx 6$ **b)** about 13.8 km/h

15. 1200 kg

16. $2 + \sqrt{n} = n$; $n = 4$

17. a) $v = \sqrt{19.6h}$, $h \geq 0$ **b)** 45.9 m

c) 34.3 m/s; A pump at 35 m/s will meet the requirements.

18. 6372.2 km

19. $a = \dfrac{3x - 4\sqrt{3x} + 4}{x}$

20. a) Example: $\sqrt{4a} = -8$

b) Example: $2 + \sqrt{x + 4} = x$

21. 2.9 m

22. 104 km

23. a) The maximum profit is \$10 000 and it requires 100 employees.

b) $n = 100 \pm \sqrt{10\,000 - P}$

c) $P \leq 10\,000$

d) domain: $n \geq 0$, $n \in W$
range: $P \leq 10\,000$, $P \in W$

24. Example: Both types of equations may involve rearranging. Solving a radical involves squaring both sides; using the quadratic formula involves taking a square root.

25. Example: Extraneous roots may occur because squaring both sides and solving the quadratic equation may result in roots that do not satisfy the original equation.