Check Your Understanding

Practise

1. Simplify each product. Identify all non-permissible values.

a) $\frac{12m^2f}{5cf} \times \frac{15c}{4m}$

b) $\frac{3(a-b)}{(a-1)(a+5)} \times \frac{(a-5)(a+5)}{15(a-b)}$

c) $\frac{(y-7)(y+3)}{(2y-3)(2y+3)} \times \frac{4(2y+3)}{(y+3)(y-1)}$

2. Write each product in simplest form. Determine all non-permissible values.

a) $\frac{d^2-100}{144} \times \frac{36}{d+10}$

b) $\frac{a+3}{a+1} \times \frac{a^2-1}{a^2-9}$

c) $\frac{4z^2-25}{2z^2-13z+20} \times \frac{z-4}{4z+10}$

d) $\frac{2p^2+5p-3}{2p-3} \times \frac{p^2-1}{6p-3} \times \frac{2p-3}{p^2+2p-3}$

3. What is the reciprocal of each rational expression?

a) $\frac{2}{t}$ **b)** $\frac{2x-1}{3}$

c) $\frac{-8}{3-y}$ **d)** $\frac{2p-3}{p-3}$

4. What are the non-permissible values in each quotient?

a) $\frac{4t^2}{3s} \div \frac{2t}{s^2}$

b) $\frac{r^2-7r}{r^2-49} \div \frac{3r^2}{r+7}$

c) $\frac{5}{n+1} \div \frac{10}{n^2-1} \div (n-1)$

5. What is the simplified product of $\frac{2x-6}{x+3}$ and $\frac{x+3}{2}$? Identify any non-permissible values.

6. What is the simplified quotient of $\frac{y^2}{y^2-9}$ and $\frac{y}{y-3}$? Identify any non-permissible values.

7. Show how to simplify each rational expression or product.

a) $\frac{3-p}{p-3}$

b) $\frac{7k-1}{3k} \times \frac{1}{1-7k}$

8. Express each quotient in simplest form. Identify all non-permissible values.

a) $\frac{2w^2-w-6}{3w+6} \div \frac{2w+3}{w+2}$

b) $\frac{v-5}{v} \div \frac{v^2-2v-15}{v^3}$

c) $\frac{9x^2-1}{x+5} \div \frac{3x^2-5x-2}{2-x}$

d) $\frac{8y^2-2y-3}{y^2-1} \div \frac{2y^2-3y-2}{2y-2} \div \frac{3-4y}{y+1}$

9. Explain why the non-permissible values in the quotient $\frac{x-5}{x+3} \div \frac{x+1}{x-2}$ are -3, -1 and 2.

Apply

10. The height of a stack of plywood is represented by $\frac{n^2-4}{n+1}$. If the number of sheets is defined by $n-2$, what expression could be used to represent the thickness of one sheet? Express your answer in simplest form.

11. Write an expression involving a product or a quotient of rational expressions for each situation. Simplify each expression.

a) The Mennonite Heritage Village in Steinbach, Manitoba, has a working windmill. If the outer end of a windmill blade turns at a rate of $\frac{x-3}{5}$ metres per minute, how far does it travel in 1 h?

b) A plane travels from Victoria to Edmonton, a distance of 900 km, in $\frac{600}{n+1}$ hours. What is the average speed of the plane?

c) Simione is shipping his carving to a buyer in Winnipeg. He makes a rectangular box with a length of $(2x - 3)$ metres and a width of $(x + 1)$ metres. The volume of the box is $(x^2 + 2x + 1)$ cubic metres. What is an expression for the height of the box?

12. How does the quotient of $\frac{3m+1}{m-1}$ and $\frac{3m+1}{m^2-1}$ compare to the quotient of $\frac{3m+1}{m^2-1}$ and $\frac{3m+1}{m-1}$? Is this always true or sometimes true? Explain your thinking.

13. Simplifying a rational expression is similar to using unit analysis to convert from one unit to another. For example, to convert 68 cm to kilometres, you can use the following steps.

$$(68\text{ cm})\left(\frac{1\text{ m}}{100\text{ cm}}\right)\left(\frac{1\text{ km}}{1000\text{ m}}\right)$$
$$= (68\ \cancel{\text{cm}}) \times \left(\frac{1\ \cancel{\text{m}}}{100\ \cancel{\text{cm}}}\right) \times \left(\frac{1\text{ km}}{1000\ \cancel{\text{m}}}\right)$$
$$= \frac{68\text{ km}}{(100)(1000)}$$
$$= 0.000\,68\text{ km}$$

Therefore, 68 cm is equivalent to 0.000 68 km. Create similar ratios that you can use to convert a measurement in yards to its equivalent in centimetres. Use 1 in. = 2.54 cm. Provide a specific example.

14. Tessa is practising for a quiz. Her work on one question is shown below.

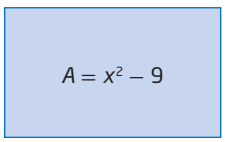
$$\frac{c^2-36}{2c} \div \frac{c+6}{8c^2}$$
$$= \frac{2c}{(c-6)(c+6)} \times \frac{c+6}{(2c)(4c)}$$
$$= \frac{\overset{1}{\cancel{2c}}}{(c-6)\underset{1}{\cancel{(c+6)}}} \times \frac{\overset{1}{\cancel{c+6}}}{\underset{1}{\cancel{(2c)}}(4c)}$$
$$= \frac{1}{4c(c-6)}$$

a) Identify any errors that Tessa made.

b) Complete the question correctly.

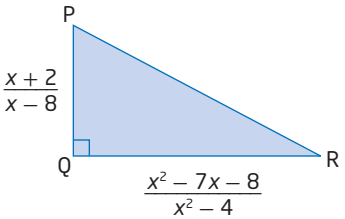
c) How does the correct answer compare with Tessa's answer? Explain.

15. Write an expression to represent the length of the rectangle. Simplify your answer.



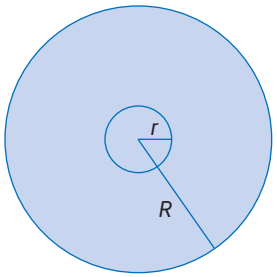
$\frac{x^2-2x-3}{x+1}$

16. What is an expression for the area of $\triangle PQR$? Give your answer in simplest form.



28. a) Lt

b)



$\pi(R - r)(R + r)$

c) $L = \dfrac{\pi(R + r)(R - r)}{t}$, $t > 0$, $R > r$, and t, R, and r should be expressed in the same units.

29. Examples:

a) $\dfrac{2}{(x + 2)(x - 5)}$

b) $\dfrac{x^2 + 3x}{x^2 + 2x - 3}$; the given expression has a non-permissible value of -1. Multiply the numerator and denominator by a factor, $x + 3$, that has a non-permissible value of -3.

30. a) Example: if $y = 7$,

$\dfrac{y - 3}{4} = \dfrac{7 - 3}{4} = 1$ and $\dfrac{2y^2 - 5y - 3}{8y + 4} = \dfrac{2(7^2) - 5(7) - 3}{8(7) + 4} = \dfrac{60}{60} = 1$

b) $\dfrac{2y^2 - 5y - 3}{8y + 4} = \dfrac{(2y + 1)(y - 3)}{4(2y + 1)} = \dfrac{y - 3}{4}$

c) The algebraic approach, in part b), proves that the expressions are equivalent for all values of y, except the non-permissible value.

31. a) $m = \dfrac{p - 8}{p + 1}$

b) Any value $-1 < p < 8$ will give a negative slope. Example: If $p = 0$, $m = \dfrac{-8}{1}$.

c) If $p = -1$, then the expression is undefined, and the line is vertical.

32. Example: $\dfrac{12}{15} = \dfrac{(3)(4)}{(3)(5)} = \dfrac{4}{5}$,

$\dfrac{x^2 - 4}{x^2 + 5x + 6} = \dfrac{(x + 2)(x - 2)}{(x + 3)(x + 2)} = \dfrac{(x - 2)}{(x + 3)}$, $x \neq -3, -2$

6.2 Multiplying and Dividing Rational Expressions, pages 327 to 330

1. a) $9m$, $c \neq 0$, $f \neq 0$, $m \neq 0$

b) $\dfrac{a - 5}{5(a - 1)}$, $a \neq -5, 1$, $a \neq b$

c) $\dfrac{4(y - 7)}{(2y - 3)(y - 1)}$, $y \neq -3, 1, \pm\dfrac{3}{2}$

2. a) $\dfrac{d - 10}{4}$, $d \neq -10$ **b)** $\dfrac{a - 1}{a - 3}$, $a \neq \pm 3, -1$

c) $\dfrac{1}{2}$, $z \neq 4, \pm\dfrac{5}{2}$

d) $\dfrac{p + 1}{3}$, $p \neq -3, 1, \dfrac{3}{2}, \dfrac{1}{2}$

3. a) $\dfrac{t}{2}$ **b)** $\dfrac{3}{2x - 1}$

c) $\dfrac{y - 3}{8}$ **d)** $\dfrac{p - 3}{2p - 3}$

4. a) $s \neq 0$, $t \neq 0$ **b)** $r \neq \pm 7, 0$

c) $n \neq \pm 1$

5. $x - 3$, $x \neq -3$

6. $\dfrac{y}{y + 3}$, $y \neq \pm 3, 0$

7. a) $\dfrac{3 - p}{p - 3} = \dfrac{-1(p - 3)}{p - 3} = -1$, $p \neq 3$

b) $\dfrac{7k - 1}{3k} \times \dfrac{1}{1 - 7k} = \dfrac{7k - 1}{3k} \times \dfrac{1}{-1(7k - 1)} = \dfrac{-1}{3k}$ or $-\dfrac{1}{3k}$, $k \neq 0, \dfrac{1}{7}$

8. a) $\dfrac{w - 2}{3}$, $w \neq -2, -\dfrac{3}{2}$

b) $\dfrac{v^2}{v + 3}$, $v \neq 0, -3, 5$,

c) $\dfrac{-1(3x - 1)}{x + 5}$, $x \neq -5, 2, -\dfrac{1}{3}$

d) $\dfrac{-2}{y - 2}$, $y \neq \pm 1, 2, -\dfrac{1}{2}, \dfrac{3}{4}$

9. -3 and -2 are the non-permissible values of the original denominators, and -1 is the non-permissible value when the reciprocal of the divisor is created.

10. $\dfrac{n^2 - 4}{n + 1} \div (n - 2)$; $\dfrac{n + 2}{n + 1}$, $n \neq -1, 2$

11. a) $\dfrac{(x - 3)}{5}(60) = 12x - 36$ metres

b) $900 \div \dfrac{600}{n + 1} = \dfrac{3n + 3}{2}$ kilometres per hour, $n \neq -1$

c) $\dfrac{x^2 + 2x + 1}{(2x - 3)(x + 1)} = \dfrac{x + 1}{2x - 3}$ metres, $x \neq \dfrac{3}{2}, -1$

12. They are reciprocals of each other. This is always true. The divisor and dividend are interchanged.

13. Example:

$1 \text{ yd}\left(\dfrac{3 \text{ ft}}{1 \text{ yd}}\right)\left(\dfrac{12 \text{ in.}}{1 \text{ ft}}\right)\left(\dfrac{2.54 \text{ cm}}{1 \text{ in.}}\right) = 91.44 \text{ cm}$

14. a) Tessa took the reciprocal of the dividend, not the divisor.

b) $= \dfrac{(c + 6)(c - 6)}{2c} \times \dfrac{8c^2}{c + 6}$

$= 4c(c - 6)$

$= 4c^2 - 24c$, $c \neq 0, -6$

c) The correct answer is the reciprocal of Tessa's answer. Taking reciprocals of either factor produces reciprocal answers.

15. $(x^2 - 9) \div \dfrac{x^2 - 2x - 3}{x + 1} = x + 3;\ x \neq 3,\ x \neq -1$

16. $\left(\dfrac{1}{2}\right)\left(\dfrac{x + 2}{x - 8}\right)\left(\dfrac{x^2 - 7x - 8}{x^2 - 4}\right);\ \dfrac{x + 1}{2(x - 2)},\ x \neq \pm 2,\ 8$

17. a) $K = \dfrac{Pw}{2h},\ m \neq 0,\ w \neq 0,\ h \neq 0$

b) $y = \dfrac{2\pi r}{x},\ d \neq 0,\ x \neq 0,\ r \neq 0$

c) $a = vw,\ w \neq 0$

18. $2(n - 4),\ \text{n} \neq -4,\ 1,\ 4$

19. a) Yes; when the two binomial factors are multiplied, you get the expression $x^2 - 5$.

b) $\dfrac{x + \sqrt{7}}{x - \sqrt{3}}$

c) $x + \sqrt{7}$; it is the same.

20. a) approximately 290 m

b) $\dfrac{(x + 3)^2}{4g(x - 5)^2}$ metres

21. Agree. Example: $\left(\dfrac{2}{3}\right)\left(\dfrac{1}{5}\right) = \dfrac{(2)(1)}{(3)(5)} = \dfrac{2}{15}$,

and $\dfrac{2}{3} \div \dfrac{1}{5} = \left(\dfrac{2}{3}\right)\left(\dfrac{5}{1}\right) = \dfrac{10}{3}$

$$\frac{(x + 2)}{(x + 3)} \times \frac{(x + 1)}{(x + 3)} = \frac{(x + 2)(x + 1)}{(x + 3)(x + 3)}$$
$$= \frac{x^2 + 3x + 2}{x^2 + 6x + 9},\ x \neq -3$$
$$\frac{(x + 2)}{(x + 3)} \div \frac{(x + 1)}{(x + 3)} = \frac{(x + 2)}{(x + 3)} \times \frac{(x + 3)}{(x + 1)}$$
$$= \frac{(x + 2)}{(x + 1)},\ x \neq -3, -1$$

22. a) $\dfrac{p + 2}{4 - p}$ **b)** $\dfrac{p - 4}{p + 2}$

23. a) $\tan \text{B} = \dfrac{b}{a}$ **b)** $\dfrac{\frac{b}{c}}{\frac{a}{c}} = \dfrac{b}{a}$

c) They are the same; $\tan \text{B} = \dfrac{\sin \text{B}}{\cos \text{B}}$.

6.3 Adding and Subtracting Rational Expressions, pages 336 to 340

1. a) $\dfrac{7x}{6}$ **b)** $\dfrac{10}{x},\ x \neq 0$

c) $\dfrac{4t + 4}{5}$ or $\dfrac{4(t + 1)}{5}$ **d)** $m,\ m \neq -1$

e) $a + 3,\ a \neq 4$

2. $$\frac{3x - 7}{9} + \frac{6x + 7}{9} = \frac{3x - 7 + 6x + 7}{9}$$
$$= \frac{9x}{9}$$
$$= x$$

3. a) $\dfrac{-4x + 13}{(x - 3)(x + 1)},\ x \neq -1,\ 3$

b) $\dfrac{3x(x + 6)}{(x - 2)(x + 10)(x + 2)},\ x \neq -10,\ \pm 2$

4. a) 24, 12; LCD = 12

b) $50a^3y^3,\ 10a^2y^2$; LCD $= 10a^2y^2$

c) $(9 - x^2)(3 + x),\ 9 - x^2$;
LCD $= 9 - x^2$ or $(3 - x)(3 + x)$

5. a) $\dfrac{11}{15a},\ a \neq 0$ **b)** $\dfrac{x + 9}{6x},\ x \neq 0$

c) $\dfrac{2(10x - 3)}{5x},\ x \neq 0$

d) $\dfrac{(2z - 3x)(2z + 3x)}{xyz},\ x \neq 0,\ y \neq 0,\ z \neq 0$

e) $\dfrac{4st + t^2 - 4}{10t^3},\ t \neq 0$

f) $\dfrac{6bxy^2 - 2ax + a^2b^2y}{a^2b^2y},\ a \neq 0,\ b \neq 0,\ y \neq 0$

6. a) $\dfrac{-5x + 18}{(x + 2)(x - 2)},\ x \neq \pm 2$

b) $\dfrac{3x - 11}{(x - 4)(x + 3)},\ x \neq -3,\ 4$

c) $\dfrac{2x(x - 4)}{(x - 2)(x + 2)},\ x \neq \pm 2$

d) $\dfrac{3}{y},\ y \neq -1,\ 0$

e) $\dfrac{-3(5h + 9)}{(h + 3)(h + 3)(h - 3)},\ h \neq \pm 3$

f) $\dfrac{(2x - 3)(x + 2)}{x(x - 2)(x - 1)(x + 3)},\ x \neq -3,\ 0,\ 1,\ 2$

7. a) $\dfrac{2(x^2 - 3x + 5)}{(x - 5)(x + 5)},\ x \neq \pm 5,\ \dfrac{1}{2}$

b) $\dfrac{-x + 4}{(x - 2)(x + 3)},\ x \neq -3,\ 0,\ 2,\ 8$

c) $\dfrac{n + 8}{(n - 4)(n - 2)},\ n \neq 2,\ 3,\ 4$

d) $\dfrac{w + 9}{(w + 3)(w + 4)},\ w \neq -2,\ -3,\ -4$

8. In the third line, multiplying by -7 should give $-7x + 14$. Also, she has forgotten to list the non-permissible values.

$$= \frac{6x + 12 + 4 - 7x + 14}{(x - 2)(x + 2)}$$
$$= \frac{-x + 30}{(x - 2)(x + 2)},\ x \neq \pm 2$$

9. Yes. Factor -1 from the numerator to create $-1(x - 5)$. Then, the expression simplifies to $\dfrac{-1}{x + 5}$.

10. a) $\dfrac{2x}{x + 3},\ x \neq 0,\ \pm 3$

b) $\dfrac{3(t + 6)}{2(t - 3)},\ t \neq -6,\ -2,\ 0,\ 3$

c) $\dfrac{3m}{m + 3},\ m \neq 0,\ -\dfrac{3}{2},\ -3$

d) $\dfrac{x}{x - 2},\ x \neq \pm 4,\ 2$