

Check Your Understanding

Practise

- Add or subtract. Express answers in simplest form. Identify any non-permissible values.
 - $\frac{11x}{6} - \frac{4x}{6}$
 - $\frac{7}{x} + \frac{3}{x}$
 - $\frac{5t+3}{10} + \frac{3t+5}{10}$
 - $\frac{m^2}{m+1} + \frac{m}{m+1}$
 - $\frac{a^2}{a-4} - \frac{a}{a-4} - \frac{12}{a-4}$
- Show that x and $\frac{3x-7}{9} + \frac{6x+7}{9}$ are equivalent expressions.
- Simplify. Identify all non-permissible values.
 - $\frac{1}{(x-3)(x+1)} - \frac{4}{(x+1)}$
 - $\frac{x-5}{x^2+8x-20} + \frac{2x+1}{x^2-4}$
- Identify two common denominators for each question. What is the LCD in each case?
 - $\frac{x-3}{6} - \frac{x-2}{4}$
 - $\frac{2}{5ay^2} + \frac{3}{10a^2y}$
 - $\frac{4}{9-x^2} - \frac{7}{3+x}$
- Add or subtract. Give answers in simplest form. Identify all non-permissible values.
 - $\frac{1}{3a} + \frac{2}{5a}$
 - $\frac{3}{2x} + \frac{1}{6}$
 - $4 - \frac{6}{5x}$
 - $\frac{4z}{xy} - \frac{9x}{yz}$
 - $\frac{2s}{5t^2} + \frac{1}{10t} - \frac{6}{15t^3}$
 - $\frac{6xy}{a^2b} - \frac{2x}{ab^2y} + 1$

- Add or subtract. Give answers in simplest form. Identify all non-permissible values.
 - $\frac{8}{x^2-4} - \frac{5}{x+2}$
 - $\frac{1}{x^2-x-12} + \frac{3}{x+3}$
 - $\frac{3x}{x+2} - \frac{x}{x-2}$
 - $\frac{5}{y+1} - \frac{1}{y} - \frac{y-4}{y^2+y}$
 - $\frac{2h}{h^2-9} + \frac{h}{h^2+6h+9} - \frac{3}{h-3}$
 - $\frac{2}{x^2+x-6} + \frac{3}{x^3+2x^2-3x}$
- Simplify each rational expression, and then add or subtract. Express answers in simplest form. Identify all non-permissible values.
 - $\frac{3x+15}{x^2-25} + \frac{4x^2-1}{2x^2+9x-5}$
 - $\frac{2x}{x^3+x^2-6x} - \frac{x-8}{x^2-5x-24}$
 - $\frac{n+3}{n^2-5n+6} + \frac{6}{n^2-7n+12}$
 - $\frac{2w}{w^2+5w+6} - \frac{w-6}{w^2+6w+8}$

Apply

- Linda has made an error in simplifying the following. Identify the error and correct the answer.

$$\begin{aligned} & \frac{6}{x-2} + \frac{4}{x^2-4} - \frac{7}{x+2} \\ &= \frac{6(x+2) + 4 - 7(x-2)}{(x-2)(x+2)} \\ &= \frac{6x+12+4-7x-14}{(x-2)(x+2)} \\ &= \frac{-x+2}{(x-2)(x+2)} \end{aligned}$$
- Can the rational expression $\frac{-x+5}{(x-5)(x+5)}$ be simplified further? Explain.

10. Simplify. State any non-permissible values.

a) $\frac{2 - \frac{6}{x}}{1 - \frac{9}{x^2}}$

b) $\frac{\frac{3}{2} + \frac{3}{t}}{\frac{t}{t+6} - \frac{1}{t}}$

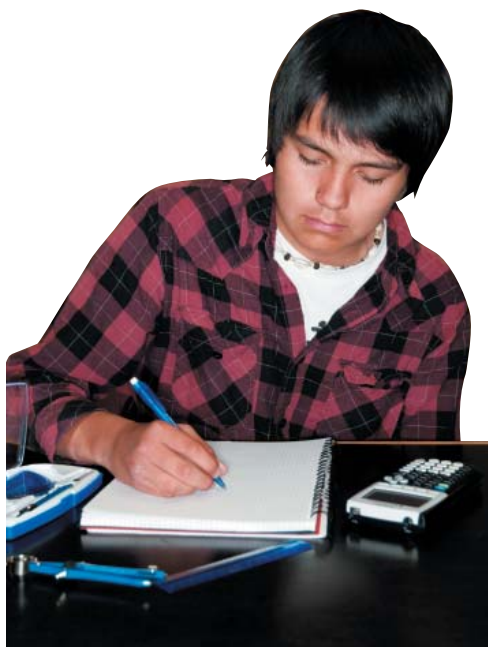
c) $\frac{\frac{3}{m} - \frac{3}{2m+3}}{\frac{3}{m^2} + \frac{1}{2m+3}}$

d) $\frac{\frac{1}{x+4} + \frac{1}{x-4}}{\frac{x}{x^2-16} + \frac{1}{x+4}}$

11. Calculators often perform calculations in a different way to accommodate the machine's logic. For each pair of rational expressions, show that the second expression is equivalent to the first one.

a) $\frac{A}{B} + \frac{C}{D}; \frac{\frac{AD}{B} + C}{D}$

b) $AB + CD + EF; \left[\frac{\left(\frac{AB}{D} + C\right)D}{F} + E \right] F$



12. A right triangle has legs of length $\frac{x}{2}$ and $\frac{x-1}{4}$. If all measurements are in the same units, what is a simplified expression for the length of the hypotenuse?

13. Ivan is concerned about an underweight calf. He decides to put the calf on a healthy growth program. He expects the calf to gain m kilograms per week and 200 kg in total. However, after some time on the program, Ivan finds that the calf has been gaining $(m + 4)$ kilograms per week.

- Explain what each of the following rational expressions tells about the situation: $\frac{200}{m}$ and $\frac{200}{m+4}$.
- Write an expression that shows the difference between the number of weeks Ivan expected to have the calf on the program and the number of weeks the calf actually took to gain 200 kg.
- Simplify your rational expression from part b). Does your simplified expression still represent the difference between the expected and actual times the calf took to gain 200 kg? Explain how you know.

14. Suppose you can type an average of n words per minute.

- What is an expression for the number of minutes it would take to type an assignment with 200 words?
- Write a sum of rational expressions to represent the time it would take you to type three assignments of 200, 500, and 1000 words, respectively.
- Simplify the sum in part b). What does the simplified rational expression tell you?
- Suppose your typing speed decreases by 5 words per minute for each new assignment. Write a rational expression to represent how much longer it would take to type the three assignments. Express your answer in simplest form.

- c) The correct answer is the reciprocal of Tessa's answer. Taking reciprocals of either factor produces reciprocal answers.

15. $(x^2 - 9) \div \frac{x^2 - 2x - 3}{x + 1} = x + 3; x \neq 3, x \neq -1$

16. $\left(\frac{1}{2}\right)\left(\frac{x+2}{x-8}\right)\left(\frac{x^2-7x-8}{x^2-4}\right); \frac{x+1}{2(x-2)}, x \neq \pm 2, 8$

17. a) $K = \frac{Pw}{2h}, m \neq 0, w \neq 0, h \neq 0$

b) $y = \frac{2\pi r}{x}, d \neq 0, x \neq 0, r \neq 0$

c) $a = vw, w \neq 0$

18. $2(n - 4), n \neq -4, 1, 4$

19. a) Yes; when the two binomial factors are multiplied, you get the expression $x^2 - 5$.

b) $\frac{x + \sqrt{7}}{x - \sqrt{3}}$

c) $x + \sqrt{7}$; it is the same.

20. a) approximately 290 m

b) $\frac{(x+3)^2}{4g(x-5)^2}$ metres

21. Agree. Example: $\left(\frac{2}{3}\right)\left(\frac{1}{5}\right) = \frac{(2)(1)}{(3)(5)} = \frac{2}{15}$,

and $\frac{2}{3} \div \frac{1}{5} = \left(\frac{2}{3}\right)\left(\frac{5}{1}\right) = \frac{10}{3}$

$$\frac{(x+2)}{(x+3)} \times \frac{(x+1)}{(x+3)} = \frac{(x+2)(x+1)}{(x+3)(x+3)}$$

$$= \frac{x^2 + 3x + 2}{x^2 + 6x + 9}, x \neq -3$$

$$\frac{(x+2)}{(x+3)} \div \frac{(x+1)}{(x+3)} = \frac{(x+2)}{(x+3)} \times \frac{(x+3)}{(x+1)}$$

$$= \frac{(x+2)}{(x+1)}, x \neq -3, -1$$

22. a) $\frac{p+2}{4-p}$

b) $\frac{p-4}{p+2}$

23. a) $\tan B = \frac{b}{a}$

b) $\frac{\frac{b}{c}}{\frac{a}{c}} = \frac{b}{a}$

c) They are the same; $\tan B = \frac{\sin B}{\cos B}$.

6.3 Adding and Subtracting Rational Expressions, pages 336 to 340

1. a) $\frac{7x}{6}$ b) $\frac{10}{x}, x \neq 0$

c) $\frac{4t+4}{5}$ or $\frac{4(t+1)}{5}$ d) $m, m \neq -1$

e) $a + 3, a \neq 4$

2. $\frac{3x-7}{9} + \frac{6x+7}{9} = \frac{3x-7+6x+7}{9}$

$$= \frac{9x}{9}$$

$$= x$$

3. a) $\frac{-4x+13}{(x-3)(x+1)}, x \neq -1, 3$

b) $\frac{3x(x+6)}{(x-2)(x+10)(x+2)}, x \neq -10, \pm 2$

4. a) 24, 12; LCD = 12

b) $50a^3y^3, 10a^2y^2$; LCD = $10a^2y^2$

c) $(9 - x^2)(3 + x), 9 - x^2$;
LCD = $9 - x^2$ or $(3 - x)(3 + x)$

5. a) $\frac{11}{15a}, a \neq 0$ b) $\frac{x+9}{6x}, x \neq 0$

c) $\frac{2(10x-3)}{5x}, x \neq 0$

d) $\frac{(2z-3x)(2z+3x)}{xyz}, x \neq 0, y \neq 0, z \neq 0$

e) $\frac{4st+t^2-4}{10t^3}, t \neq 0$

f) $\frac{6bxy^2 - 2ax + a^2b^2y}{a^2b^2y}, a \neq 0, b \neq 0, y \neq 0$

6. a) $\frac{-5x+18}{(x+2)(x-2)}, x \neq \pm 2$

b) $\frac{3x-11}{(x-4)(x+3)}, x \neq -3, 4$

c) $\frac{2x(x-4)}{(x-2)(x+2)}, x \neq \pm 2$

d) $\frac{3}{y}, y \neq -1, 0$

e) $\frac{-3(5h+9)}{(h+3)(h+3)(h-3)}, h \neq \pm 3$

f) $\frac{(2x-3)(x+2)}{x(x-2)(x-1)(x+3)}, x \neq -3, 0, 1, 2$

7. a) $\frac{2(x^2-3x+5)}{(x-5)(x+5)}, x \neq \pm 5, \frac{1}{2}$

b) $\frac{-x+4}{(x-2)(x+3)}, x \neq -3, 0, 2, 8$

c) $\frac{n+8}{(n-4)(n-2)}, n \neq 2, 3, 4$

d) $\frac{w+9}{(w+3)(w+4)}, w \neq -2, -3, -4$

8. In the third line, multiplying by -7 should give $-7x + 14$. Also, she has forgotten to list the non-permissible values.

$$= \frac{6x+12+4-7x+14}{(x-2)(x+2)}$$

$$= \frac{-x+30}{(x-2)(x+2)}, x \neq \pm 2$$

9. Yes. Factor -1 from the numerator to create $-1(x - 5)$. Then, the expression simplifies to $\frac{-1}{x+5}$.

10. a) $\frac{2x}{x+3}, x \neq 0, \pm 3$

b) $\frac{3(t+6)}{2(t-3)}, t \neq -6, -2, 0, 3$

c) $\frac{3m}{m+3}, m \neq 0, -\frac{3}{2}, -3$

d) $\frac{x}{x-2}, x \neq \pm 4, 2$

$$\begin{aligned}
 11. \text{ a) } \frac{\frac{AD}{B} + C}{D} &= \left(\frac{AD + CB}{B} \right) \div D \\
 &= \left(\frac{AD + CB}{B} \right) \left(\frac{1}{D} \right) \\
 &= \frac{AD + CB}{BD} \\
 &= \frac{AD}{BD} + \frac{CB}{BD} \\
 &= \frac{A}{B} + \frac{C}{D}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } \left[\frac{\left(\frac{AB}{D} + C \right) D}{F} + E \right] F &= \left(\frac{AB}{D} + C \right) D + EF \\
 &= AB + CD + EF
 \end{aligned}$$

$$12. \frac{\sqrt{5x^2 - 2x + 1}}{4}$$

13. a) $\frac{200}{m}$ tells the expected number of weeks to gain 200 kg; $\frac{200}{m+4}$ tells the number of weeks to gain 200 kg when the calf is on the healthy growth program.

$$\text{b) } \frac{200}{m} - \frac{200}{m+4}$$

c) $\frac{800}{m(m+4)}$, $m \neq 0, -4$; yes, the expressions are equivalent.

14. a) $\frac{200}{n}$ minutes

$$\text{b) } \left(\frac{200}{n} + \frac{500}{n} + \frac{1000}{n} \right) \text{ minutes}$$

c) $\frac{1700}{n}$ minutes; the time it would take to type all three assignments

$$\begin{aligned}
 \text{d) } \left(\frac{200}{n} + \frac{500}{n-5} + \frac{1000}{n-10} \right) - \frac{1700}{n} \\
 = \frac{12\,500n - 75\,000}{n(n-5)(n-10)}
 \end{aligned}$$

15. a) $\frac{2x^2 + 13}{(x-4)(x+5)}$, $x \neq -5, -2, 0, 3, 4$

$$\text{b) } \frac{-9}{(x-1)(x+2)}, x \neq -3, -2, 0, 1, \frac{1}{2}$$

$$\text{c) } \frac{3(1-4x)}{(x+5)(x-4)}, x \neq -5, -2, 0, 3, 4$$

$$\text{d) } \frac{15}{(x+6)(x+3)}, x \neq 0, -2, -3, -6, -\frac{1}{2}$$

16. $\left(\frac{20}{x} + \frac{16}{x-2} \right)$ hours

17. Example: In a three-person relay, Barry ran the first 12 km at a constant rate. Jim ran the second leg of 8 km at a rate 3 km/h faster, and Al ran the last leg of 5 km at a rate 2 km/h slower than Barry. The total time for the relay would be $\left(\frac{12}{x} + \frac{8}{x+3} + \frac{5}{x-2} \right)$ hours.

18. a) Incorrect: $\frac{a}{b} - \frac{b}{a} = \frac{a^2 - b^2}{ab}$. Find the LCD first, do not just combine pieces.

b) Incorrect: $\frac{ca + cb}{c + cd} = \frac{a + b}{1 + d}$. Factor c from the numerator and from the denominator, remembering that $c(1) = c$.

c) Incorrect: $\frac{a}{4} - \frac{6-b}{4} = \frac{a-6+b}{4}$. Distribute the subtraction to both terms in the numerator of the second rational expression by first putting the numerator in brackets.

d) Incorrect: $\frac{1}{1 - \frac{a}{b}} = \frac{b}{b-a}$. Simplify the denominator first, and then divide.

e) Incorrect: $\frac{1}{a-b} = \frac{-1}{b-a}$. Multiplying both numerator and denominator by -1 , which is the same as multiplying the whole expression by 1, changes every term to its opposite.

19. a) Agree. Each term in the numerator is divided by the denominator, and then can be simplified.

b) Disagree. If Keander was given the rational expression $\frac{3x-7}{x}$, there are multiple original expressions that he could come up with, for example $\frac{2x-1}{x} + \frac{x-6}{x}$ or $\frac{x^2 - x + 11}{x} - \frac{x^2 - 4x + 18}{x}$.

$$20. \text{ a) } \frac{12}{13} \Omega$$

$$\text{b) } \frac{R_1 R_2 R_3}{R_2 R_3 + R_1 R_3 + R_1 R_2}$$

$$\text{c) } \frac{12}{13} \Omega$$

d) the simplified form from part b), because with it you do not need to find the LCD first

21. Example:

Arithmetic:	Algebra:
If $\frac{2}{3} = \frac{6}{9}$, then	If $\frac{x}{2} = \frac{3x}{6}$, then
$\frac{2}{3} = \frac{2-6}{3-9}$	$\frac{x}{2} = \frac{x-3x}{2-6}$
$= \frac{-4}{-6}$	$= \frac{-2x}{-4}$
$= \frac{2}{3}$	$= \frac{x}{2}$

22. a) $\frac{-2p+9}{2(p-3)}$, $p \neq 3$

b) $\frac{3}{0}$; the slope is undefined when $p = 3$, so this is a vertical line through A and B.

c) The slope is negative.

d) When $p = 4$, the slope is positive; from $p = 5$ to $p = 10$ the slope is always negative.