Key Ideas

- You can solve a rational equation by multiplying both sides by a common denominator. This eliminates the fractions from the equation. Then, solve the resulting equation.
- When solving a word problem involving rates, it is helpful to use a table.
- Check that the potential roots satisfy the original equation, are not non-permissible values, and, in the case of a word problem, are realistic in the context.

Check Your Understanding

Practise

1. Use the LCD to eliminate the fractions from each equation. Do not solve.

a) $\frac{x-1}{3} - \frac{2x-5}{4} = \frac{5}{12} + \frac{x}{6}$

b) $\frac{2x+3}{x+5} + \frac{1}{2} = \frac{7}{2x+10}$

c) $\frac{4x}{x^2-9} - \frac{5}{x+3} = 2$

2. Solve and check each equation. Identify all non-permissible values.

a) $\frac{f+3}{2} - \frac{f-2}{3} = 2$

b) $\frac{3-y}{3y} + \frac{1}{4} = \frac{1}{2y}$

c) $\frac{9}{w-3} - \frac{4}{w-6} = \frac{18}{w^2-9w+18}$

3. Solve each rational equation. Identify all non-permissible values.

a) $\frac{6}{t} + \frac{t}{2} = 4$

b) $\frac{6}{c-3} = \frac{c+3}{c^2-9} - 5$

c) $\frac{d}{d+4} = \frac{2-d}{d^2+3d-4} + \frac{1}{d-1}$

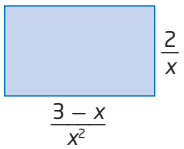
d) $\frac{x^2+x+2}{x+1} - x = \frac{x^2-5}{x^2-1}$

4. Joline solved the following rational equation. She claims that the solution is $y = 1$. Do you agree? Explain.

$$\frac{-3y}{y-1} + 6 = \frac{6y-9}{y-1}$$

Apply

5. A rectangle has the dimensions shown.



a) What is an expression for the difference between the length and the width of the rectangle? Simplify your answer.

b) What is an expression for the area of the rectangle? Express the answer in simplest form.

c) If the perimeter of the rectangle is 28 cm, find the value(s) for x.

6. Solve. Round answers to the nearest hundredth.

a) $\frac{26}{b+5} = 1 + \frac{3}{b-2}$

b) $\frac{c}{c+2} - 3 = \frac{-6}{c^2-4}$

7. Experts claim that the golden rectangle is most pleasing to the eye. It has dimensions that satisfy the equation $\frac{l}{w} = \frac{l+w}{l}$, where w is the width and l is the length.

According to this relationship, how long should a rectangular picture frame be if its width is 30 cm? Give the exact answer and an approximate answer, rounded to the nearest tenth of a centimetre.

8. The sum of two numbers is 25. The sum of their reciprocals is $\frac{1}{4}$. Determine the two numbers.

9. Two consecutive numbers are represented by x and $x + 1$. If 6 is added to the first number and two is subtracted from the second number, the quotient of the new numbers is $\frac{9}{2}$. Determine the numbers algebraically.

10. A French club collected the same amount from each student going on a trip to Le Cercle Molière in Winnipeg. When six students could not go, each of the remaining students was charged an extra $3. If the total cost was $540, how many students went on the trip?

Did You Know?

Le Cercle Molière is the oldest continuously running theatre company in Canada, founded in 1925. It is located in St. Boniface, Manitoba, and moved into its new building in 2009.

11. The sum of the reciprocals of two consecutive integers is $\frac{11}{30}$. What are the integers?

12. Suppose you are running water into a tub. The tub can be filled in 2 min if only the cold tap is used. It fills in 3 min if only the hot tap is turned on. How long will it take to fill the tub if both taps are on simultaneously?

a) Will the answer be less than or greater than 2 min? Why?

b) Complete a table in your notebook similar to the one shown.

	Time to Fill Tub (min)	Fraction Filled in 1 min	Fraction Filled in x minutes
Cold Tap			
Hot Tap			
Both Taps	x	$\frac{1}{x}$	$\frac{x}{x}$ or 1

c) What is one equation that represents both taps filling the tub?

d) Solve your equation to determine the time with both taps running.

13. Two hoses together fill a pool in 2 h. If only hose A is used, the pool fills in 3 h. How long would it take to fill the pool if only hose B were used?

14. Two kayakers paddle 18 km downstream with the current in the same time it takes them to go 8 km upstream against the current. The rate of the current is 3 km/h.

a) Complete a table like the following in your notebook. Use the formula distance = rate × time.

	Distance (km)	Rate (km/h)	Time (h)
Downstream			
Upstream			

b) What equation could you use to find the rate of the kayakers in still water?

c) Solve your equation.

d) Which values are non-permissible?

Did You Know?

When you are travelling with the current, add the speed of the current to your rate of speed. When you are travelling against the current, subtract the speed of the current.

Kyuquot Sound, British Columbia

23. 3

24. Examples: $\frac{2}{5} + \frac{1}{5} = \frac{2+1}{5} = \frac{3}{5}$ and

$\frac{2}{5} + \frac{1}{3} = \frac{2(3)+1(5)}{15} = \frac{11}{15}$

$\frac{2}{x} + \frac{1}{x} = \frac{2+1}{x} = \frac{3}{x}$ and

$\frac{2}{x} + \frac{1}{y} = \frac{2(y)+1(x)}{xy} = \frac{2y+x}{xy}$

25. a) The student's suggestion is correct.

Example: find the average of $\frac{1}{2}$ and $\frac{3}{4}$.

$\left(\frac{1}{2} + \frac{3}{4}\right) \div 2 = \left(\frac{2+3}{4}\right) \times \left(\frac{1}{2}\right)$

$= \frac{5}{8}$

Halfway between $\frac{1}{2}$ and $\frac{3}{4}$, or $\frac{4}{8}$ and $\frac{6}{8}$, is $\frac{5}{8}$.

b) $\frac{13}{4a}$, $a \neq 0$

26. Yes. Example: $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ and $\frac{1}{2} + \frac{1}{3} = \frac{1}{\frac{6}{5}} = \frac{5}{6}$

$\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy}$ and $\frac{1}{x} + \frac{1}{y} = \frac{1}{\frac{xy}{x+y}} = \frac{x+y}{xy}$

27. a) $\frac{1}{u} + \frac{1}{v} = \frac{u+v}{uv}$ **b)** 5.93 cm

c) $f = \frac{uv}{u+v}$

28. Step 3 Yes

Step 4 a) $A = 2, B = 1$

b) $A = 3, B = 3$

Step 5 Always:

$\frac{3}{x-4} + \frac{-2}{x-1} = \frac{3(x-1)+-2(x-4)}{(x-4)(x-1)}$

$= \frac{x+5}{(x-4)(x-1)}$

6.4 Rational Equations, pages 348 to 351

1. a) $4(x-1) - 3(2x-5) = 5 + 2x$

b) $2(2x+3) + 1(x+5) = 7$

c) $4x - 5(x-3) = 2(x+3)(x-3)$

2. a) $f = -1$ **b)** $y = 6, y \neq 0$

c) $w = 12, w \neq 3, 6$

3. a) $t = 2$ or $t = 6, t \neq 0$ **b)** $c = 2, c \neq \pm 3$

c) $d = -2$ or $d = 3, d \neq -4, 1$

d) $x = 3, x \neq \pm 1$

4. No. The solution is not a permissible value.

5. a) $\frac{3-x}{x^2} - \frac{2}{x}, \frac{3-3x}{x^2}, x > 0$

b) $\frac{3-x}{x^2} \times \frac{2}{x}, \frac{6-2x}{x^3}, x > 0$

c) $x = \frac{1}{2}$

6. a) $b = 3.44$ or $b = 16.56$

b) $c = -3.54$ or $c = 2.54$

7. $l = 15(\sqrt{5} + 1)$, 48.5 cm

8. The numbers are 5 and 20.

9. The numbers are 3 and 4.

10. 30 students

11. The integers are 5 and 6.

12. a) Less than 2 min. There is more water going in at once.

b)

	Time to Fill Tub (min)	Fraction Filled in 1 min	Fraction Filled in x minutes
Cold Tap	2	$\frac{1}{2}$	$\frac{x}{2}$
Hot Tap	3	$\frac{1}{3}$	$\frac{x}{3}$
Both Taps	x	$\frac{1}{x}$	1

c) $\frac{x}{2} + \frac{x}{3} = 1$ **d)** 1.2 min

13. 6 h

14. a)

	Distance (km)	Rate (km/h)	Time (h)
Downstream	18	$x+3$	$\frac{18}{x+3}$
Upstream	8	$x-3$	$\frac{8}{x-3}$

b) $\frac{18}{x+3} = \frac{8}{x-3}$ **c)** 7.8 km/h

d) $x \neq \pm 3$

15. 28.8 h

16. 5.7 km/h

17. about 50 km/h west of Swift Current, and 60 km/h east of Swift Current

18. about 3.5 km/h

19.

	Reading Rate in Pages per Day	Number of Pages Read	Number of Days
First Half	x	259	$\frac{259}{x}$
Second Half	$x+12$	259	$\frac{259}{x+12}$

about 20 pages per day for the first half of the book

20. a) 2 L **b)** 4.5 L

21. $a = \pm\frac{1}{3}$

22. a) $\frac{\frac{1}{a}+\frac{1}{b}}{2} = \frac{1}{x}, x = \frac{2ab}{a+b}$

b) 4 and 12, or -6 and 2

23. a)

$\frac{1}{x} - \frac{1}{y} = a$

$y - x = axy$

$y = axy + x$

$y = x(ay+1)$

$\frac{y}{ay+1} = x$

or

$\frac{1}{x} - \frac{1}{y} = a$

$\frac{y-x}{xy} = a$

$y - x = axy$

$y = axy + x$

$y = x(ay+1)$

$\frac{y}{ay+1} = x$

In both, $x \neq 0$, $y \neq 0$, $ay \neq -1$.