Check Your Understanding

Practise

1. Write each expression with base 2.

a) 4^6 **b)** 8^3

c) $\left(\frac{1}{8}\right)^2$ **d)** 16

2. Rewrite the expressions in each pair so that they have the same base.

a) 2^3 and 4^2

b) 9^x and 27

c) $\left(\frac{1}{2}\right)^{2x}$ and $\left(\frac{1}{4}\right)^{x-1}$

d) $\left(\frac{1}{8}\right)^{x-2}$ and 16^x

3. Write each expression as a single power of 4.

a) $(\sqrt{16})^2$ **b)** $\sqrt[3]{16}$

c) $\sqrt{16}\left(\sqrt[3]{64}\right)^2$ **d)** $(\sqrt{2})^8(\sqrt[4]{4})^4$

4. Solve. Check your answers using substitution.

a) $2^{4x} = 4^{x+3}$ **b)** $25^{x-1} = 5^{3x}$

c) $3^{w+1} = 9^{w-1}$ **d)** $36^{3m-1} = 6^{2m+5}$

5. Solve. Check your answers using graphing technology.

a) $4^{3x} = 8^{x-3}$ **b)** $27^x = 9^{x-2}$

c) $125^{2y-1} = 25^{y+4}$ **d)** $16^{2k-3} = 32^{k+3}$

6. Solve for x using systematic trial. Check your answers using graphing technology. Round answers to one decimal place.

a) $2 = 1.07^x$ **b)** $3 = 1.1^x$

c) $0.5 = 1.2^{x-1}$ **d)** $5 = 1.08^{x+2}$

7. Solve for t graphically. Round answers to two decimal places, if necessary.

a) $100 = 10(1.04)^t$ **b)** $10 = \left(\frac{1}{2}\right)^{2t}$

c) $12 = \left(\frac{1}{4}\right)^{\frac{t}{3}}$ **d)** $100 = 25\left(\frac{1}{2}\right)^{\frac{t}{4}}$

e) $2^t = 3^{t-1}$ **f)** $5^{t-2} = 4^t$

g) $8^{t+1} = 3^{t-1}$ **h)** $7^{2t+1} = 4^{t-2}$

Apply

8. If seafood is not kept frozen (below 0 °C), it will spoil due to bacterial growth. The relative rate of spoilage increases with temperature according to the model $R = 100(2.7)^{\frac{T}{8}}$, where T is the temperature, in degrees Celsius, and R is the relative spoilage rate.

a) Sketch a graph of the relative spoilage rate R versus the temperature T from 0 °C to 25 °C.

b) Use your graph to predict the temperature at which the relative spoilage rate doubles to 200.

c) What is the relative spoilage rate at 15 °C?

d) If the maximum acceptable relative spoilage rate is 500, what is the maximum storage temperature?

Did You Know?

The relative rate of spoilage for seafood is defined as the shelf life at 0° C divided by the shelf life at temperature T, in degrees Celsius.

9. A bacterial culture starts with 2000 bacteria and doubles every 0.75 h. After how many hours will the bacteria count be 32 000?

10. Simionie needs $7000 to buy a snowmobile, but only has $6000. His bank offers a GIC that pays an annual interest rate of 3.93%, compounded annually. How long would Simionie have to invest his money in the GIC to have enough money to buy the snowmobile?

Did You Know?

A Guaranteed Investment Certificate (GIC) is a secure investment that guarantees 100% of the original amount that is invested. The investment earns interest, at either a fixed or a variable rate, based on a predetermined formula.

C1 Example: The graph of an exponential function of the form $y = c^x$ has a horizontal asymptote at $y = 0$. Since $y \neq 0$, the graph cannot have an x-intercept.

C2 a) Example: For a function of the form $y = a(c)^{b(x-h)} + k$, the parameters a and k can affect the x-intercept. If $a > 0$ and $k < 0$ or $a < 0$ and $k > 0$, then the graph of the exponential function will have an x-intercept.

b) Example: For a function of the form $y = a(c)^{b(x-h)} + k$, the parameters a, h, and k can affect the y-intercept. The point $(0, y)$ on the graph of $y = c^x$ gets mapped to $(h, ay + k)$.

7.3 Solving Exponential Equations, pages 364 to 365

1. a) 2^{12} **b)** 2^9 **c)** 2^{-6} **d)** 2^4

2. a) 2^3 and 2^4 **b)** 3^{2x} and 3^3

c) $\left(\frac{1}{2}\right)^{2x}$ and $\left(\frac{1}{2}\right)^{2x-2}$ **d)** 2^{-3x+6} and 2^{4x}

3. a) 4^2 **b)** $4^{\frac{2}{3}}$ **c)** 4^3 **d)** 4^3

4. a) $x = 3$ **b)** $x = -2$ **c)** $w = 3$ **d)** $m = \frac{7}{4}$

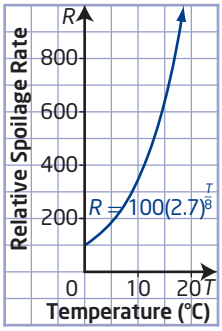
5. a) $x = -3$ **b)** $x = -4$ **c)** $y = \frac{11}{4}$ **d)** $k = 9$

6. a) 10.2 **b)** 11.5 **c)** −2.8 **d)** 18.9

7. a) 58.71 **b)** −1.66 **c)** −5.38 **d)** −8

e) 2.71 **f)** 14.43 **g)** −3.24 **h)** −1.88

8. a)



b) approximately 5.6 °C

c) approximately 643

d) approximately 13.0 °C

9. 3 h

10. 4 years

11. a) $A = 1000(1.02)^n$ **b)** $1372.79 **c)** 9 years

12. a) $C = \left(\frac{1}{2}\right)^{\frac{t}{5.3}}$ **b)** $\frac{1}{32}$ of the original amount

c) 47.7 years

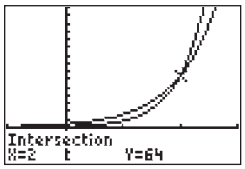
13. a) $A = 500(1.033)^n$ **b)** $691.79

c) approximately 17 years

14. $5796.65

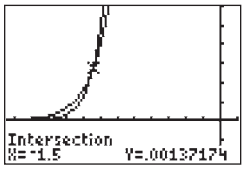
15. a) i) $x > 2$ **ii)** $x > -\frac{3}{2}$

b) i)



Since the graph of $y = 2^{3x}$ is greater than (above) the graph of $y = 4^{x+1}$ when $x > 2$, the solution is $x > 2$.

ii)



Since the graph of $y = 81^x$ is less than (below) the graph of $y = 27^{2x+1}$ when $x > -\frac{3}{2}$, the solution is $x > -\frac{3}{2}$.

c) Example: Solve the inequality $\left(\frac{1}{2}\right)^{x+3} > 2^{x-1}$.

Answer: $x < -1$

16. Yes. Rewrite the equation as $(4^x)^2 + 2(4^x) - 3 = 0$ and factor as $(4^x + 3)(4^x - 1) = 0$; $x = 0$

17. $(2^x)^x = \left(2^{\frac{5}{2}}\right)^{\frac{5}{2}} \approx 76.1$

18. 20 years

C1 a) You can express 16^2 with a base of 4 by writing 16 as 4^2 and simplifying.

$16^2 = (4^2)^2$

$16^2 = 4^4$

b) Example: You can express 16^2 with a base of 2 by writing 16 as 2^4 and simplifying.

$16^2 = (2^4)^2$

$16^2 = 2^8$

Or, you can express 16^2 with a base of $\frac{1}{4}$ by writing 16 as $\left(\frac{1}{4}\right)^{-2}$ and simplifying.

$16^2 = \left(\left(\frac{1}{4}\right)^{-2}\right)^2$

$16^2 = \left(\frac{1}{4}\right)^{-4}$

C2 a)

$16^{2x} = 8^{x-3}$

$(2^4)^{2x} = (2^3)^{x-3}$

$2^{8x} = 2^{3x-9}$

$8x = 3x - 9$

$5x = -9$

$x = -\frac{9}{5}$

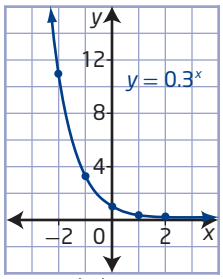
b) Step 1: Express the bases on both sides as powers of 2.
Step 2: Apply the power of a power law.
Step 3: Equate the exponents.
Step 4: Isolate the term containing x.
Step 5: Solve for x.

Chapter 7 Review, pages 366 to 367

1. a) B **b)** D **c)** A **d)** C

2. a)

x	y
−2	$11.\overline{1}$
−1	$3.\overline{3}$
0	1
1	0.3
2	0.09



b) domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > 0, y \in \mathrm{R}\}$, y-intercept 1, function decreasing, horizontal asymptote $y = 0$

3. $y = \left(\frac{1}{4}\right)^x$

4. a) Since the interest rate is 3.25% per year, each year the investment grows by a factor of 103.25%, which, written as a decimal, is 1.0325.

b) $1.38 **c)** 21.7 years

5. a) $a = -2$: vertical stretch by a factor of 2 and reflection in the x-axis; $b = 3$: horizontal stretch by a factor of $\frac{1}{3}$; $h = 1$: horizontal translation of 1 unit right; $k = 2$: vertical translation of 2 units up

b)

Transformation	Parameter Value	Function Equation
horizontal stretch	$b = 3$	$y = 4^{3x}$
vertical stretch	$a = -2$	$y = -2(4)^x$
translation left/right	$h = 1$	$y = (4)^{x-1}$
translation up/down	$k = 2$	$y = 4^x + 2$