Check Your Understanding

Practise

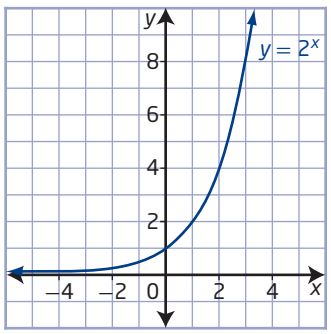
1. For each exponential graph,

i) copy the graph on grid paper, and then sketch the graph of the inverse on the same grid

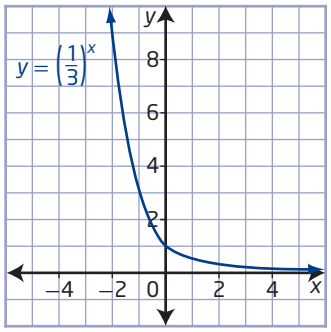
ii) write the equation of the inverse

iii) determine the following characteristics of the inverse graph:
- the domain and range
- the x-intercept, if it exists
- the y-intercept, if it exists
- the equation of the asymptote

a)



b)



2. Express in logarithmic form.

a) $12^2 = 144$ **b)** $8^{\frac{1}{3}} = 2$

c) $10^{-5} = 0.000\ 01$ **d)** $7^{2x} = y + 3$

3. Express in exponential form.

a) $\log_5 25 = 2$

b) $\log_8 4 = \frac{2}{3}$

c) $\log 1\ 000\ 000 = 6$

d) $\log_{11} (x + 3) = y$

4. Use the definition of a logarithm to evaluate.

a) $\log_5 125$

b) $\log 1$

c) $\log_4 \sqrt[3]{4}$

d) $\log_{\frac{1}{3}} 27$

5. Without using technology, find two consecutive whole numbers, a and b, such that $a < \log_2 28 < b$.

6. State a value of x so that $\log_3 x$ is

a) a positive integer

b) a negative integer

c) zero

d) a rational number

7. The base of a logarithm can be any positive real number except 1. Use examples to illustrate why the base of a logarithm cannot be

a) 0

b) 1

c) negative

8. a) If $f(x) = 5^x$, state the equation of the inverse, $f^{-1}(x)$.

b) Sketch the graph of $f(x)$ and its inverse. Identify the following characteristics of the inverse graph:
- the domain and range
- the x-intercept, if it exists
- the y-intercept, if it exists
- the equations of any asymptotes

9. a) If $g(x) = \log_{\frac{1}{4}} x$, state the equation of the inverse, $g^{-1}(x)$.

b) Sketch the graph of $g(x)$ and its inverse. Identify the following characteristics of the inverse graph:
- the domain and range
- the x-intercept, if it exists
- the y-intercept, if it exists
- the equations of any asymptotes

Apply

10. Explain the relationship between the characteristics of the functions $y = 7^x$ and $y = \log_7 x$.

11. Graph $y = \log_2 x$ and $y = \log_{\frac{1}{2}} x$ on the same coordinate grid. Describe the ways the graphs are

a) alike

b) different

12. Determine the value of x in each.

a) $\log_6 x = 3$

b) $\log_x 9 = \frac{1}{2}$

c) $\log_{\frac{1}{4}} x = -3$

d) $\log_x 16 = \frac{4}{3}$

13. Evaluate each expression.

a) 5^m, where $m = \log_5 7$

b) 8^n, where $n = \log_8 6$

14. Evaluate.

a) $\log_2 (\log_3 (\log_4 64))$

b) $\log_4 (\log_2 (\log 10^{16}))$

15. Determine the x-intercept of $y = \log_7 (x + 2)$.

16. The point $\left(\frac{1}{8}, -3\right)$ is on the graph of the logarithmic function $f(x) = \log_c x$, and the point $(4, k)$ is on the graph of the inverse, $y = f^{-1}(x)$. Determine the value of k.

17. The growth of a new social networking site can be modelled by the exponential function $N(t) = 1.1^t$, where N is the number of users after t days.

a) Write the equation of the inverse.

b) How long will it take, to the nearest day, for the number of users to exceed 1 000 000?

18. The Palermo Technical Impact Hazard scale was developed to rate the potential hazard impact of a near-Earth object. The Palermo scale, P, is defined as $P = \log R$, where R is the relative risk. Compare the relative risks of two asteroids, one with a Palermo scale value of -1.66 and the other with a Palermo scale value of -4.83.

19. The formula for the Richter magnitude, M, of an earthquake is $M = \log \frac{A}{A_0}$, where A is the amplitude of the ground motion and A_0 is the amplitude of a standard earthquake. In 1985, an earthquake with magnitude 6.9 on the Richter scale was recorded in the Nahanni region of the Northwest Territories. The largest recorded earthquake in Saskatchewan occurred in 1982 near the town of Big Beaver. It had a magnitude of 3.9 on the Richter scale. How many times as great as the seismic shaking of the Saskatchewan earthquake was that of the Nahanni earthquake?

Did You Know?

Scientists at the Geological Survey of Canada office, near Sidney, British Columbia, record and locate earthquakes every day. Of the approximately 1000 earthquakes each year in Western Canada, fewer than 50 are strong enough to be felt by humans. In Canada, there have been no casualties directly related to earthquakes. A tsunami triggered by a major earthquake off the coast of California could be hazardous to the British Columbia coast.

20. If $\log_5 x = 2$, then determine $\log_5 125x$.

Extend

21. If $\log_3 (m - n) = 0$ and $\log_3 (m + n) = 3$, determine the values of m and n.

22. If $\log_3 m = n$, then determine $\log_3 m^4$, in terms of n.

23. Determine the equation of the inverse of $y = \log_2 (\log_3 x)$.

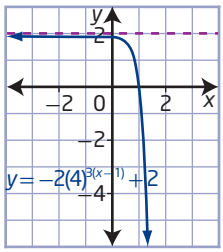
24. If $m = \log_2 n$ and $2m + 1 = \log_2 16n$, determine the values of m and n.

Create Connections

C1 Graph $y = |\log_2 x|$. Describe how the graph of $y = |\log_2 x|$ is related to the graph of $y = \log_2 x$.

C2 Create a mind map to summarize everything you know about the graph of the logarithmic function $y = \log_c x$, where $c > 0$ and $c \neq 1$. Enhance your mind map by sharing ideas with classmates.

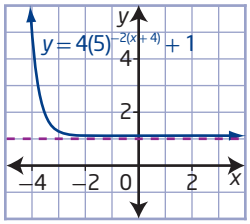
c)

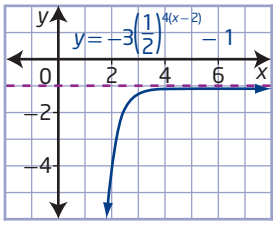


d) domain $\{x \mid x \in \mathbb{R}\}$, range $\{y \mid y < 2, y \in \mathbb{R}\}$, horizontal asymptote $y = 2$, y-intercept $\frac{63}{32}$, x-intercept 1

6. a) horizontal translation of 3 units right
b) vertical translation of 4 units down
c) reflection in the x-axis and a translation of 1 unit left and 2 units up

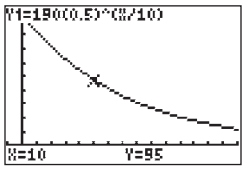
7. a) $y = 4(5)^{-2(x+4)} + 1$ **b)** $y = -3\left(\frac{1}{2}\right)^{4(x-2)} - 1$





8. a) $a = 190$: vertical stretch by a factor of 190; $b = \frac{1}{10}$: horizontal stretch by a factor of 10

b)



c) domain $\{t \mid t \geq 0, t \in \mathbb{R}\}$, range $\{T \mid 0 < T \leq 190, T \in \mathbb{R}\}$

d) approximately 31.1 h

9. a) 6^2 **b)** 6^{-2} **c)** 6^5

10. a) $x = -\frac{3}{2}$ **b)** $x = \frac{12}{11}$

11. a) $x \approx -4.30$ **b)** $x \approx -6.13$

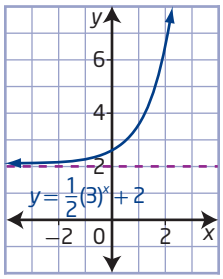
12. a) $N = \left(\frac{1}{2}\right)^{\frac{t}{2.5}}$ **b)** $\frac{1}{16}$ **c)** 25 h

Chapter 7 Practice Test, pages 368 to 369

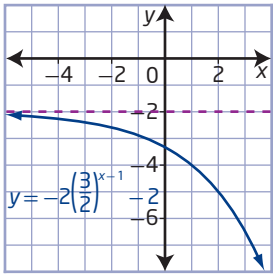
1. B **2.** C **3.** B **4.** A **5.** D

6. a) $y = 5^{x+3} + 2$ **b)** $y = -0.5(2)^{x-1} - 4$

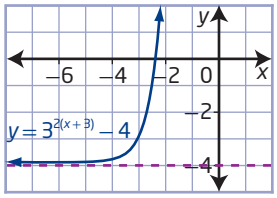
7. a)



b)

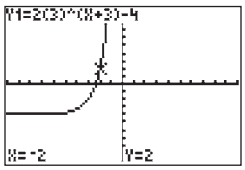


c)



8. a) $a = 2$: vertical stretch by a factor of 2; $h = -3$: horizontal translation of 3 units left; $k = -4$: vertical translation of 4 units down

b)



c) domain $\{x \mid x \in \mathbb{R}\}$, range $\{y \mid y > -4, y \in \mathbb{R}\}$, horizontal asymptote $y = -4$

9. a) $x = -4$ **b)** $x = 18$ **c)** $x = \frac{13}{8}$

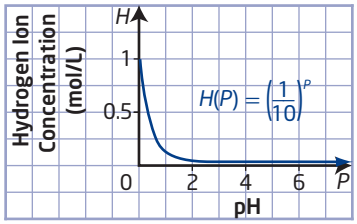
10. a) $x \approx 9.7$ **b)** $x \approx 0.1$

11. a) 1.0277 **b)** $P = 100(1.0277)^t$

c) domain $\{t \mid t \geq 0, t \in \mathbb{R}\}$, range $\{P \mid P \geq 100, P \in \mathbb{R}\}$

d) approximately 8.2 years

12. a)



b) 1.0×10^{-7} [H^+]

c) 1.0×10^{-7} to 2.5×10^{-8} [H^+]

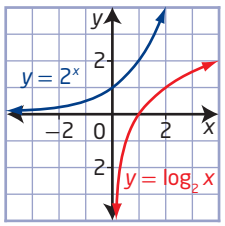
13. 4.5 years

14. 9.97 years

Chapter 8 Logarithmic Functions

8.1 Understanding Logarithms, pages 380 to 382

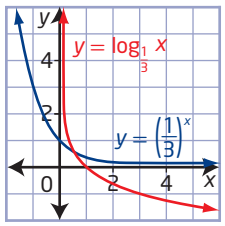
1. a) i)



ii) $y = \log_2 x$

iii) domain $\{x \mid x > 0, x \in \mathbb{R}\}$, range $\{y \mid y \in \mathbb{R}\}$, x-intercept 1, no y-intercept, vertical asymptote $x = 0$

b) i)



ii) $y = \log_{\frac{1}{3}} x$

iii) domain $\{x \mid x > 0, x \in \mathbb{R}\}$, range $\{y \mid y \in \mathbb{R}\}$, x-intercept 1, no y-intercept, vertical asymptote $x = 0$

2. a) $\log_{12} 144 = 2$ **b)** $\log_8 2 = \frac{1}{3}$

c) $\log_{10} 0.000\,01 = -5$ **d)** $\log_7 (y + 3) = 2x$

3. a) $5^2 = 25$ **b)** $8^{\frac{2}{3}} = 4$

c) $10^6 = 1\,000\,000$ **d)** $11^y = x + 3$

4. a) 3 **b)** 0 **c)** $\frac{1}{3}$ **d)** -3

5. $a = 4$; $b = 5$

6. a) $x > 1$ **b)** $0 < x < 1$ **c)** $x = 1$

d) Example: $x = 9$

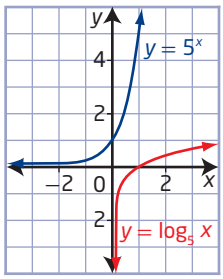
7. a) 0 raised to any non-zero power is 0.

b) 1 raised to any power is 1.

c) Exponential functions with a negative base are not continuous.

8. a) $y = \log_5 x$

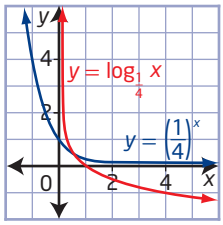
b)



domain $\{x \mid x > 0, x \in \mathrm{R}\}$,
range $\{y \mid y \in \mathrm{R}\}$,
x-intercept 1,
no y-intercept,
vertical asymptote $x = 0$

9. a) $g^{-1}(x) = \left(\frac{1}{4}\right)^x$

b)



domain $\{x \mid x \in \mathrm{R}\}$,
range $\{y \mid y > 0, y \in \mathrm{R}\}$,
no x-intercept,
y-intercept 1,
horizontal asymptote $y = 0$

10. They are reflections of each other in the line $y = x$.

11. a) They have the exact same shape.

b) One of them is increasing and the other is decreasing.

12. a) 216 **b)** 81 **c)** 64 **d)** 8

13. a) 7 **b)** 6

14. a) 0 **b)** 1

15. -1

16. 16

17. a) $t = \log_{1.1} N$ **b)** 145 days

18. The larger asteroid had a relative risk that was 1479 times as dangerous.

19. 1000 times as great

20. 5

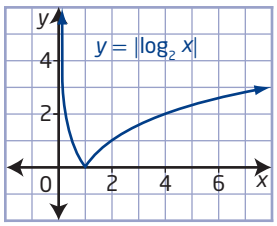
21. $m = 14$, $n = 13$

22. $4n$

23. $y = 3^{2^x}$

24. $n = 8$; $m = 3$

C1



The function has the same general shape, but instead of decreasing, after $x = 1$ the function increases without limit.

C2 Answers will vary.

C3 **Step 1: a)** $e = 2.718\,281\,828$ **b)** 10^{10}

Step 2: a) domain $\{x \mid x > 0, x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$, x-intercept 1, no y-intercept, vertical asymptote $x = 0$

b) $y = \ln x$

Step 3: a) $r = 2.41$

b) i) $\theta = \frac{\ln r}{0.14}$ **ii)** $\theta = 17.75$

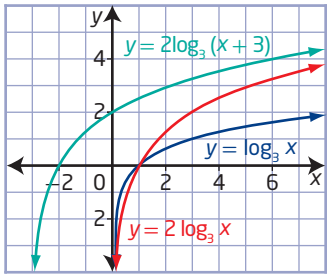
8.2 Transformations of Logarithmic Functions, pages 389 to 391

1. a) Translate 1 unit right and 6 units up.

b) Reflect in the x-axis, stretch vertically about the x-axis by a factor of 4, and stretch horizontally about the y-axis by a factor of $\frac{1}{3}$.

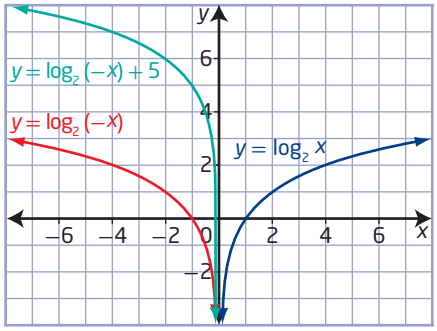
c) Reflect in the y-axis, stretch vertically about the x-axis by a factor of $\frac{1}{2}$, and translate 7 units up.

2. a)



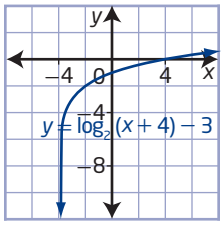
b) $y = 2 \log_3 (x + 3)$

3. a)

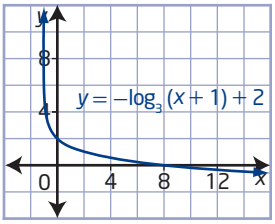


b) $y = \log_2 (-x) + 5$

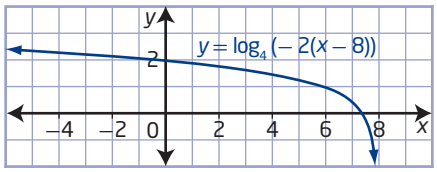
4. a)



b)



c)



5. a) **i)** vertical asymptote $x = -3$

ii) domain $\{x \mid x > -3, x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

iii) y-intercept -5 **iv)** x-intercept -2

b) **i)** vertical asymptote $x = -9$

ii) domain $\{x \mid x > -9, x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

iii) y-intercept 2 **iv)** x-intercept -8.75

c) **i)** vertical asymptote $x = -3$

ii) domain $\{x \mid x > -3, x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

iii) y-intercept -1.3 **iv)** x-intercept 22

d) **i)** vertical asymptote $x = -1$

ii) domain $\{x \mid x > -1, x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

iii) y-intercept -6 **iv)** x-intercept $-\frac{3}{4}$