Key Ideas

- Let P be any real number, and M, N, and c be positive real numbers with $c \neq 1$. Then, the following laws of logarithms are valid.

Name	Law	Description
Product	$\log_c MN = \log_c M + \log_c N$	The logarithm of a product of numbers is the sum of the logarithms of the numbers.
Quotient	$\log_c \frac{M}{N} = \log_c M - \log_c N$	The logarithm of a quotient of numbers is the difference of the logarithms of the dividend and divisor.
Power	$\log_c M^P = P \log_c M$	The logarithm of a power of a number is the exponent times the logarithm of the number.

- Many quantities in science are measured using a logarithmic scale. Two commonly used logarithmic scales are the decibel scale and the pH scale.

Check Your Understanding

Practise

1. Write each expression in terms of individual logarithms of x, y, and z.

a) $\log_7 xy^3\sqrt{z}$

b) $\log_5 (xyz)^8$

c) $\log \frac{x^2}{y\sqrt[3]{z}}$

d) $\log_3 x\sqrt{\frac{y}{z}}$

2. Use the laws of logarithms to simplify and evaluate each expression.

a) $\log_{12} 24 - \log_{12} 6 + \log_{12} 36$

b) $3 \log_5 10 - \frac{1}{2} \log_5 64$

c) $\log_3 27\sqrt{3}$

d) $\log_2 72 - \frac{1}{2}(\log_2 3 + \log_2 27)$

3. Write each expression as a single logarithm in simplest form.

a) $\log_9 x - \log_9 y + 4 \log_9 z$

b) $\frac{\log_3 x}{2} - 2 \log_3 y$

c) $\log_6 x - \frac{1}{5}(\log_6 x + 2 \log_6 y)$

d) $\frac{\log x}{3} + \frac{\log y}{3}$

4. The original use of logarithms was to simplify calculations. Use the approximations shown on the right and the laws of logarithms to perform each calculation using only paper and pencil.

a) 1.44×1.2 $\quad \log 1.44 \approx 0.158\ 36$

b) $1.728 \div 1.2$ $\quad \log 1.2 \approx 0.079\ 18$

c) $\sqrt{1.44}$ $\quad \log 1.728 \approx 0.237\ 54$

5. Evaluate.

a) 3^k, where $k = \log_2 40 - \log_2 5$

b) 7^n, where $n = 3 \log_8 4$

Apply

6. To obtain the graph of $y = \log_2 8x$, you can either stretch or translate the graph of $y = \log_2 x$.

a) Describe the stretch you need to apply to the graph of $y = \log_2 x$ to result in the graph of $y = \log_2 8x$.

b) Describe the translation you need to apply to the graph of $y = \log_2 x$ to result in the graph of $y = \log_2 8x$.

7. Decide whether each equation is true or false. Justify your answer. Assume c, x, and y are positive real numbers and $c \neq 1$.

a) $\dfrac{\log_c x}{\log_c y} = \log_c x - \log_c y$

b) $\log_c (x + y) = \log_c x + \log_c y$

c) $\log_c c^n = n$

d) $(\log_c x)^n = n \log_c x$

e) $-\log_c \left(\dfrac{1}{x}\right) = \log_c x$

8. If $\log 3 = P$ and $\log 5 = Q$, write an algebraic expression in terms of P and Q for each of the following.

a) $\log \dfrac{3}{5}$

b) $\log 15$

c) $\log 3\sqrt{5}$

d) $\log \dfrac{25}{9}$

9. If $\log_2 7 = K$, write an algebraic expression in terms of K for each of the following.

a) $\log_2 7^6$

b) $\log_2 14$

c) $\log_2 (49 \times 4)$

d) $\log_2 \dfrac{\sqrt[5]{7}}{8}$

10. Write each expression as a single logarithm in simplest form. State any restrictions on the variable.

a) $\log_5 x + \log_5 \sqrt{x^3} - 2 \log_5 x$

b) $\log_{11} \dfrac{x}{\sqrt{x}} + \log_{11} \sqrt{x^5} - \dfrac{7}{3} \log_{11} x$

11. Write each expression as a single logarithm in simplest form. State any restrictions on the variable.

a) $\log_2 (x^2 - 25) - \log_2 (3x - 15)$

b) $\log_7 (x^2 - 16) - \log_7 (x^2 - 2x - 8)$

c) $2 \log_8 (x + 3) - \log_8 (x^2 + x - 6)$

12. Show that each equation is true for $c > 0$ and $c \neq 1$.

a) $\log_c 48 - (\log_c 3 + \log_c 2) = \log_c 8$

b) $7 \log_c 4 = 14 \log_c 2$

c) $\dfrac{1}{2}(\log_c 2 + \log_c 6) = \log_c 2 + \log_c \sqrt{3}$

d) $\log_c (5c)^2 = 2(\log_c 5 + 1)$

13. Sound intensity, β, in decibels is defined as $\beta = 10 \log \left(\dfrac{I}{I_0}\right)$, where I is the intensity of the sound measured in watts per square metre (W/m^2) and I_0 is 10^{-12} W/m^2, the threshold of hearing.

a) The sound intensity of a hairdryer is 0.000 01 W/m^2. Find its decibel level.

b) A fire truck siren has a decibel level of 118 dB. City traffic has a decibel level of 85 dB. How many times as loud as city traffic is the fire truck siren?

c) The sound of Elly's farm tractor is 63 times as intense as the sound of her car. If the decibel level of the car is 80 dB, what is the decibel level of the farm tractor?

14. Abdi incorrectly states, "A noise of 20 dB is twice as loud as a noise of 10 dB." Explain the error in Abdi's reasoning.

15. The term *decibel* is also used in electronics for current and voltage ratios. Gain is defined as the ratio between the signal coming in and the signal going out. The gain, G, in decibels, of an amplifier is defined as $G = 20 \log \dfrac{V}{V_i}$, where V is the voltage output and V_i is the voltage input. If the gain of an amplifier is 24 dB when the voltage input is 0.2 V, find the voltage output, V. Answer to the nearest tenth of a volt.

6. a) $y = 5 \log x$ **b)** $y = \log_8 2x$

c) $y = \frac{1}{3} \log_2 x$ **d)** $y = \log_4 \left(\frac{x}{2}\right)$

7. a) stretch horizontally about the y-axis by a factor of $\frac{1}{4}$; translate 5 units left and 6 units up

b) stretch horizontally about the y-axis by a factor of 3; stretch vertically about the x-axis by a factor of 2; reflect in the y-axis; translate 1 unit right and 4 units down

8. a) $a = -1, b = 1, h = -6, k = 3$; $y = -\log_3 (x + 6) + 3$

b) $a = 5, b = 3, h = 0, k = 0$; $y = 5 \log_3 3x$

c) $a = 0.75, b = -0.25, h = 2, k = -5$; $y = \frac{3}{4} \log_3 \left(-\frac{1}{4}(x - 2)\right) - 5$

9. a) Reflect in the y-axis, stretch vertically about the x-axis by a factor of 5, stretch horizontally about the y-axis by a factor of $\frac{1}{4}$, and translate 3 units right and 2 units down.

b) Reflect in the x-axis, reflect in the y-axis, stretch vertically about the x-axis by a factor of $\frac{1}{4}$, translate 6 units right and 1 unit up.

10. a) $y = \log_3 x - 6$ **b)** $y = \log_2 \left(\frac{x}{4}\right)$

11. Stretch vertically about the x-axis by a factor of 3 and translate 4 units right and 2 units down.

12. a) Stretch vertically about the x-axis by a factor of 0.67, stretch horizontally about the y-axis by a factor of $\frac{25}{9}$ or approximately 2.78, and translate 1.46 units up.

b) 515 649 043 kWh

13. a) 0.8 µL **b)** 78 mmHg

14. a) 172 cm **b)** 40 kg

15. $a = \frac{1}{3}$

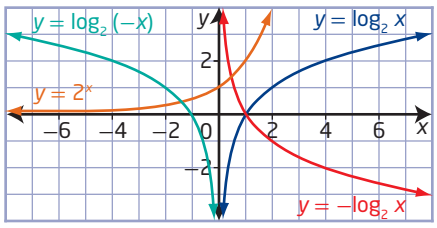
16. a) $y = -2 \log_5 x + 13$ **b)** $y = \log 2x$

17. $a = \frac{1}{2}, k = -8$

C1 $a = \frac{1}{4}, b = \frac{1}{3}, h = 4, k = -1$; $g(x) = 0.25 \log_5 \left(\frac{1}{3}\right)(x - 4) - 1$

C2 a) $y = -\log_2 x$, $y = \log_2 (-x)$, $y = 2^x$

b) Reflect in the x-axis, reflect in the y-axis, and reflect in the line $y = x$.



C3 a) $y = \frac{1}{2} \log_7 \frac{(x - 5)}{3} + \frac{1}{2}$ **b)** $y = 3^{\frac{x-8}{2}} + 1$

C4 Answers will vary.

8.3 Laws of Logarithms, pages 400 to 403

1. a) $\log_7 x + 3 \log_7 y + \frac{1}{2} \log_7 z$

b) $8(\log_5 x + \log_5 y + \log_5 z)$

c) $2 \log x - \log y - \frac{1}{3} \log z$

d) $y = \log_3 x + \left(\frac{1}{2}\right)(\log_3 y - \log_3 z)$

2. a) 2 **b)** 3 **c)** 3.5 **d)** 3

3. a) $\log_9 \left(\frac{xz^4}{y}\right)$ **b)** $y = \log_3 \frac{\sqrt{x}}{y^2}$

c) $\log_6 \left(\frac{x}{\sqrt[3]{xy^2}}\right)$ **d)** $\log \sqrt[3]{xy}$

4. a) 1.728 **b)** 1.44 **c)** 1.2

5. a) 27 **b)** 49

6. a) Stretch horizontally about the y-axis by a factor of $\frac{1}{8}$.

b) Translate 3 units up.

7. a) False; the division must take place inside the logarithm.

b) False; it must be a multiplication inside the logarithm.

c) True

d) False; the power must be inside the logarithm.

e) True

8. a) $P - Q$ **b)** $P + Q$ **c)** $P + \frac{Q}{2}$ **d)** $2Q - 2P$

9. a) $6K$ **b)** $1 + K$ **c)** $2K + 2$ **d)** $\frac{K}{5} - 3$

10. a) $\frac{1}{2} \log_5 x, x > 0$ **b)** $\frac{2}{3} \log_{11} x, x > 0$

11. a) $\log_2 \left(\frac{x + 5}{3}\right), x < -5$ or $x > 5$

b) $\log_7 \left(\frac{x + 4}{x + 2}\right), x < -4$ or $x > 4$

c) $\log_8 \left(\frac{x + 3}{x - 2}\right), x > 2$

12. a) Left Side $= \log_c 48 - (\log_c 3 + \log_c 2)$
$= \log_c 48 - \log_c 6$
$= \log_c 8$
$=$ Right Side

b) Left Side $= 7 \log_c 4$
$= 7 \log_c 2^2$
$= 2(7) \log_c 2$
$= 14 \log_c 2$
$=$ Right Side

c) Left Side $= \frac{1}{2}(\log_c 2 + \log_c 6)$
$= \frac{1}{2}(\log_c 2 + \log_c 3 + \log_c 2)$
$= \frac{1}{2}(2 \log_c 2) + \frac{1}{2} \log_c 3$
$= \log_c 2 + \log_c \sqrt{3}$
$=$ Right Side

d) Left Side $= \log_c (5c)^2$
$= 2 \log_c 5c$
$= 2 (\log_c 5 + \log_c c)$
$= 2 (\log_c 5 + 1)$
$=$ Right Side

13. a) 70 dB **b)** approximately 1995 times as loud

c) approximately 98 dB

14. Decibels must be changed to intensity to gauge loudness. The function that maps the change is not linear.

15. 3.2 V

16. a) 10^{-7} mol/L **b)** 12.6 times as acidic **c)** 3.4

17. 0.18 km/s

18. a) The graphs are the same for $x > 0$. However, the graph of $y = \log x^2$ has a second branch for $x < 0$, which is the reflection in the y-axis of the branch for $x > 0$.

b) The domains are different. The function $y = \log x^2$ is defined for all values of x except 0, while the function $y = 2 \log x$ is defined only for $x > 0$.

c) $x > 0$