Check Your Understanding

Practise

1. a) State the five key points for $y = \sin x$ that occur in one complete cycle from 0 to 2π.

b) Use the key points to sketch the graph of $y = \sin x$ for $-2\pi \leq x \leq 2\pi$. Indicate the key points on your graph.

c) What are the x-intercepts of the graph?

d) What is the y-intercept of the graph?

e) What is the maximum value of the graph? the minimum value?

2. a) State the five key points for $y = \cos x$ that occur in one complete cycle from 0 to 2π.

b) Use the key points to sketch a graph of $y = \cos x$ for $-2\pi \leq x \leq 2\pi$. Indicate the key points on your graph.

c) What are the x-intercepts of the graph?

d) What is the y-intercept of the graph?

e) What is the maximum value of the graph? the minimum value?

3. Copy and complete the table of properties for $y = \sin x$ and $y = \cos x$ for all real numbers.

Property	$y = \sin x$	$y = \cos x$
maximum		
minimum		
amplitude		
period		
domain		
range		
y-intercept		
x-intercepts		

4. State the amplitude of each periodic function. Sketch the graph of each function.

a) $y = 2 \sin \theta$ **b)** $y = \frac{1}{2} \cos \theta$

c) $y = -\frac{1}{3} \sin x$ **d)** $y = -6 \cos x$

5. State the period for each periodic function, in degrees and in radians. Sketch the graph of each function.

a) $y = \sin 4\theta$ **b)** $y = \cos \frac{1}{3}\theta$

c) $y = \sin \frac{2}{3}x$ **d)** $y = \cos 6x$

Apply

6. Match each function with its graph.

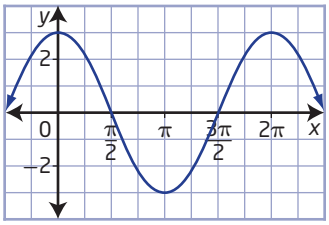
a) $y = 3 \cos x$

b) $y = \cos 3x$

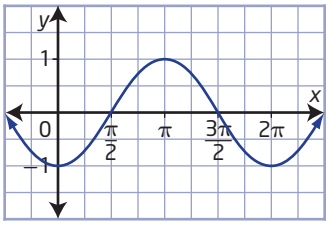
c) $y = -\sin x$

d) $y = -\cos x$

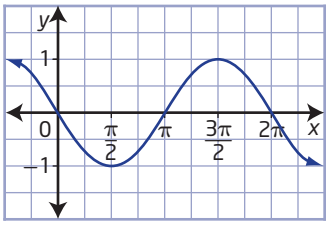
A



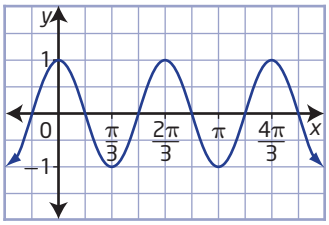
B



C



D



7. Determine the amplitude of each function. Then, use the language of transformations to describe how each graph is related to the graph of $y = \sin x$.

a) $y = 3 \sin x$ **b)** $y = -5 \sin x$

c) $y = 0.15 \sin x$ **d)** $y = -\frac{2}{3} \sin x$

8. Determine the period (in degrees) of each function. Then, use the language of transformations to describe how each graph is related to the graph of $y = \cos x$.

a) $y = \cos 2x$ **b)** $y = \cos (-3x)$

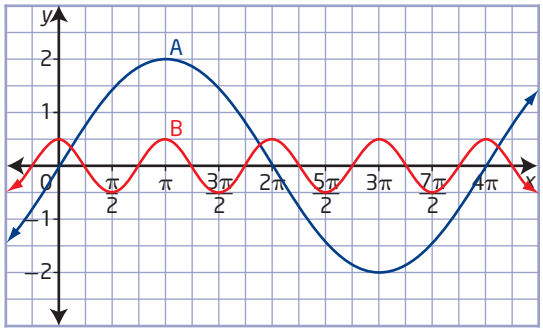
c) $y = \cos \frac{1}{4}x$ **d)** $y = \cos \frac{2}{3}x$

9. Without graphing, determine the amplitude and period of each function. State the period in degrees and in radians.

a) $y = 2 \sin x$ **b)** $y = -4 \cos 2x$

c) $y = \frac{5}{3} \sin \left(-\frac{2}{3}x\right)$ **d)** $y = 3 \cos \frac{1}{2}x$

10. a) Determine the period and the amplitude of each function in the graph.



b) Write an equation in the form $y = a \sin bx$ or $y = a \cos bx$ for each function.

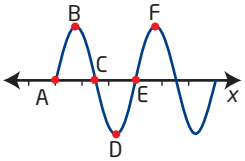
c) Explain your choice of either sine or cosine for each function.

11. Sketch the graph of each function over the interval $[-360°, 360°]$. For each function, clearly label the maximum and minimum values, the x-intercepts, the y-intercept, the period, and the range.

a) $y = 2 \cos x$ **b)** $y = -3 \sin x$

c) $y = \frac{1}{2} \sin x$ **d)** $y = -\frac{3}{4} \cos x$

12. The points indicated on the graph shown represent the x-intercepts and the maximum and minimum values.



a) Determine the coordinates of points B, C, D, and E if $y = 3 \sin 2x$ and A has coordinates (0, 0).

b) Determine the coordinates of points C, D, E, and F if $y = 2 \cos x$ and B has coordinates (0, 2).

c) Determine the coordinates of points B, C, D, and E if $y = \sin \frac{1}{2}x$ and A has coordinates $(-4\pi, 0)$.

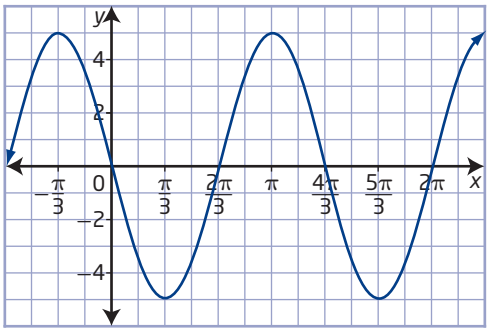
13. The second harmonic in sound is given by $f(x) = \sin 2x$, while the third harmonic is given by $f(x) = \sin 3x$. Sketch the curves and compare the graphs of the second and third harmonics for $-2\pi \leq x \leq 2\pi$.

Did You Know?

A harmonic is a wave whose frequency is an integral multiple of the fundamental frequency. The fundamental frequency of a periodic wave is the inverse of the period length.

14. Sounds heard by the human ear are vibrations created by different air pressures. Musical sounds are regular or periodic vibrations. Pure tones will produce single sine waves on an oscilloscope. Determine the amplitude and period of each single sine wave shown.

a)



c) In the expression $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$, substitute the y-value for the opposite side and 1 for the hypotenuse. Since $x^2 + y^2 = 1$ then $\cos^2\theta + \sin^2\theta = 1$. Substitute the value you determined for $\sin\theta$ into $\cos^2\theta + \sin^2\theta = 1$ and solve for $\cos\theta$.

8. a) Cosine is negative in quadrants II and III. Find the reference angle by subtracting π from the given angle in quadrant III. To find the solution in quadrant II, subtract the reference angle from π.

b) Given each solution θ, add $2\pi n$, $n \in \mathrm{I}$ to obtain each general solution $\theta + 2\pi n$, $n \in \mathrm{I}$.

9. $\theta = \dfrac{3\pi}{4} + 2\pi n$, $n \in \mathrm{I}$ or $\theta = \dfrac{5\pi}{4} + 2\pi n$, $n \in \mathrm{I}$

10. Since $1° = \dfrac{\pi}{180}$, then $3° = \dfrac{3\pi}{180}$ or $\dfrac{\pi}{60}$.

$3 = \dfrac{3(180°)}{\pi} \approx 172°$.

11. a) quadrant III **b)** 40°

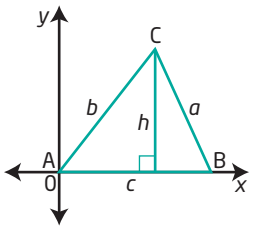
c) $\sin(-500°) = -0.6$, $\cos(-500°) = -0.8$, $\tan(-500°) = 0.8$, $\csc(-500°) = -1.6$, $\sec(-500°) = -1.3$, $\cot(-500°) = 1.2$

12. a) $\dfrac{5\pi}{4}, -\dfrac{3\pi}{4}; \dfrac{5\pi}{4} \pm 2\pi n$, $n \in \mathrm{N}$

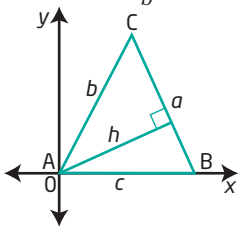
b) $145°, -215°, 145° \pm (360°)n$, $n \in \mathrm{N}$

13. 7.7 km

14.



Given $A = \frac{1}{2}bh$, b = side c, since $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ then $\sin A = \frac{h}{b}$ or $h = b \sin A$ and $A = \frac{1}{2}bc \sin A$ or



Given $A = \frac{1}{2}bh$, b = side a, since $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ then $\sin B = \frac{h}{c}$ or $h = c \sin B$, therefore $A = \frac{1}{2}ac \sin B$.

15. a) $\theta = -\dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{7\pi}{4}, -2.21, 0.93, 4.07$

b) 0.67, 2.48 **c)** $0, \pi, 2\pi, 4.47, 1.33$

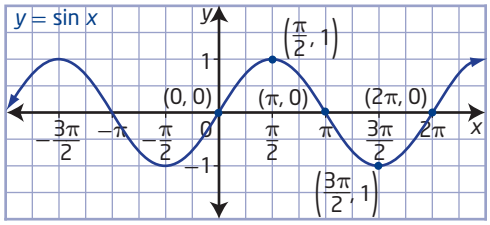
16. $\dfrac{28\pi}{3}$ m or 29.32 m

Chapter 5 Trigonometric Functions and Graphs

5.1 Graphing Sine and Cosine Functions, pages 233 to 237

1. a) $(0, 0), \left(\frac{\pi}{2}, 1\right), (\pi, 0), \left(\frac{3\pi}{2}, -1\right), (2\pi, 0)$

b)



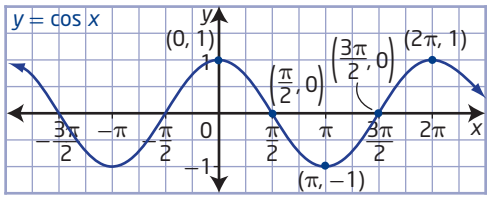
c) x-intercepts: $-2\pi, -\pi, 0, \pi, 2\pi$

d) y-intercept: 0

e) The maximum value is 1, and the minimum value is -1.

2. a) $(0, 1), \left(\frac{\pi}{2}, 0\right), (\pi, -1), \left(\frac{3\pi}{2}, 0\right), (2\pi, 1)$

b)



c) x-intercepts: $-\dfrac{3\pi}{2}, -\dfrac{\pi}{2}, \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

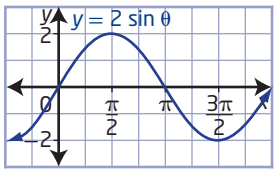
d) y-intercept: 1

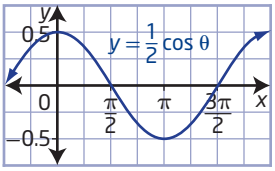
e) The maximum value is 1, and the minimum value is -1.

3.

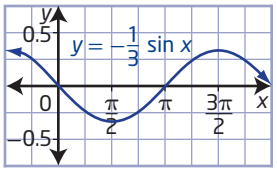
Property	$y = \sin x$	$y = \cos x$
maximum	1	1
minimum	-1	-1
amplitude	1	1
period	2π	2π
domain	$\{x \mid x \in \mathrm{R}\}$	$\{x \mid x \in \mathrm{R}\}$
range	$\{y \mid -1 \le y \le 1, y \in \mathrm{R}\}$	$\{y \mid -1 \le y \le 1, y \in \mathrm{R}\}$
y-intercept	0	1
x-intercepts	$\pi n, n \in \mathrm{I}$	$\frac{\pi}{2} + \pi n, n \in \mathrm{I}$

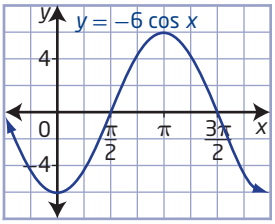
4. a) 2 **b)** $\frac{1}{2}$



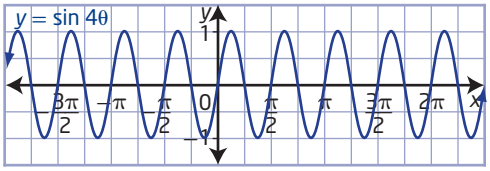


c) $\frac{1}{3}$ **d)** 6

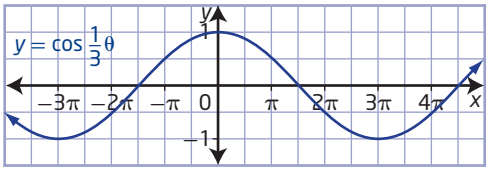




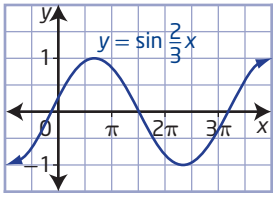
5. a) $\frac{\pi}{2}$ or 90°



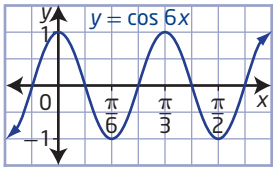
b) 6π or 1080°



c) 3π or 540°



d) $\frac{\pi}{3}$ or 60°



6. a) A **b)** D **c)** C **d)** B

7. a) Amplitude is 3; stretched vertically by a factor of 3 about the x-axis.

b) Amplitude is 5; stretched vertically by a factor of 5 about the x-axis and reflected in the x-axis.

c) Amplitude is 0.15; stretched vertically by a factor of 0.15 about the x-axis.

d) Amplitude is $\frac{2}{3}$; stretched vertically by a factor of $\frac{2}{3}$ about the x-axis and reflected in the x-axis.

8. a) Period is 180°; stretched horizontally by a factor of $\frac{1}{2}$ about the y-axis.

b) Period is 120°; stretched horizontally by a factor of $\frac{1}{3}$ about the y-axis and reflected in the y-axis.

c) Period is 1440°; stretched horizontally by a factor of 4 about the y-axis.

d) Period is 540°; stretched horizontally by a factor of $\frac{3}{2}$ about the y-axis.

9. a) Amplitude is 2; period is 360° or 2π.

b) Amplitude is 4; period is 180° or π.

c) Amplitude is $\frac{5}{3}$; period is 540° or 3π.

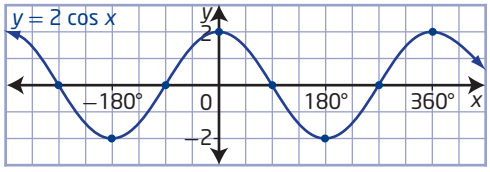
d) Amplitude is 3; period is 720° or 4π.

10. a) Graph A: Amplitude is 2 and period is 4π. Graph B: Amplitude is 0.5 and period is π.

b) Graph A: $y = 2 \sin \frac{1}{2}x$; Graph B: $y = 0.5 \cos 2x$

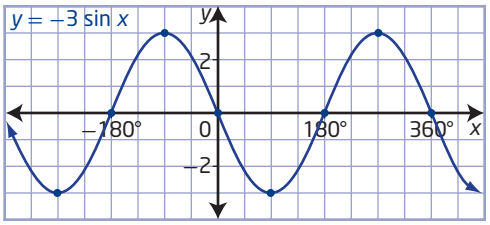
c) Graph A starts at 0, so the sine function is the obvious choice. Graph B starts at 1, so the cosine function is the obvious choice.

11. a)



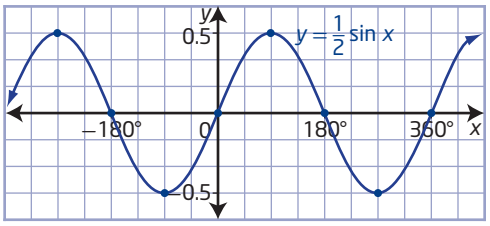
Property	Points on the Graph of $y = 2 \cos x$
maximum	(−360°, 2), (0°, 2), (360°, 2)
minimum	(−180°, −2), (180°, −2)
x-intercepts	(−270°, 0), (−90°, 0), (90°, 0), (270°, 0)
y-intercept	(0, 2)
period	360°
range	$\{y \mid -2 \le y \le 2, y \in R\}$

b)



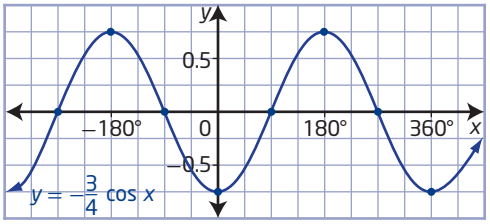
Property	Points on the Graph of $y = -3 \sin x$
maximum	(−90°, 3), (270°, 3)
minimum	(−270°, −3), (90°, −3)
x-intercepts	(−360°, 0), (−180°, 0), (0°, 0), (180°, 0), (360°, 0)
y-intercept	(0, 0)
period	360°
range	$\{y \mid -3 \le y \le 3, y \in R\}$

c)



Property	Points on the Graph of $y = \frac{1}{2} \sin x$
maximum	(−270°, 0.5), (90°, 0.5)
minimum	(−90°, −0.5), (270°, −0.5)
x-intercepts	(−360°, 0), (−180°, 0), (0°, 0), (180°, 0), (360°, 0)
y-intercept	(0, 0)
period	360°
range	$\{y \mid -0.5 \le y \le 0.5, y \in R\}$

d)



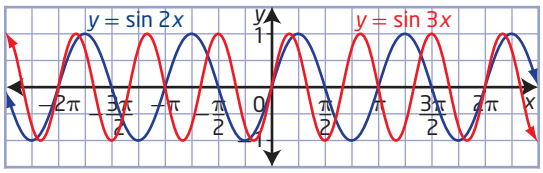
Property	Points on the Graph of $y = -\frac{3}{4} \cos x$
maximum	(−180°, 0.75), (180°, 0.75)
minimum	(−360°, −0.75), (0°, −0.75), (360°, −0.75)
x-intercepts	(−270°, 0), (−90°, 0), (90°, 0), (270°, 0)
y-intercept	(0, −0.75)
period	360°
range	$\{y \mid -0.75 \le y \le 0.75, y \in R\}$

12. a) $B\left(\frac{\pi}{4}, 3\right)$, $C\left(\frac{\pi}{2}, 0\right)$, $D\left(\frac{3\pi}{4}, -3\right)$, $E(\pi, 0)$

b) $C\left(\frac{\pi}{2}, 0\right)$, $D(\pi, -2)$, $E\left(\frac{3\pi}{2}, 0\right)$, $F(2\pi, 2)$

c) $B(-3\pi, 1)$, $C(-2\pi, 0)$, $D(-\pi, -1)$, $E(0, 0)$

13.



The amplitude, maximum, minimum, y-intercepts, domain, and range are the same for both graphs. The period and x-intercepts are different.

14. a) Amplitude is 5; period is $\frac{4\pi}{3}$.

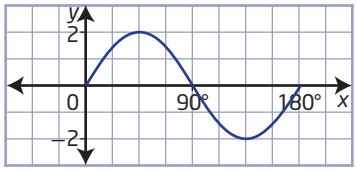
b) Amplitude is 4; Period is $\frac{2\pi}{3}$.

15. a) Amplitude is 20 mm Hg; Period is 0.8 s.

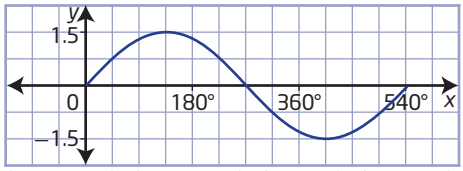
b) 75 bpm

16. Answers may vary.

17. a)



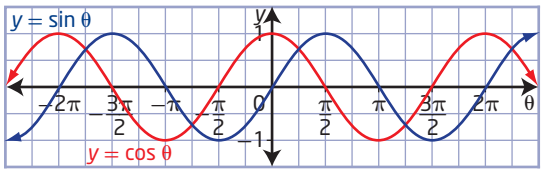
b)



18. a) $\left(-\frac{7\pi}{4}, \frac{\sqrt{2}}{2}\right)$, $\left(-\frac{5\pi}{4}, \frac{\sqrt{2}}{2}\right)$, $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$, $\left(\frac{9\pi}{4}, \frac{\sqrt{2}}{2}\right)$;
Find the points of intersection of $y = \sin\theta$ and $y = \frac{\sqrt{2}}{2}$.

b) $\left(-\frac{11\pi}{6}, \frac{\sqrt{3}}{2}\right)$, $\left(-\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right)$, $\left(\frac{11\pi}{6}, \frac{\sqrt{3}}{2}\right)$, $\left(\frac{13\pi}{6}, \frac{\sqrt{3}}{2}\right)$;
Find the points of intersection of $y = \cos\theta$ and $y = \frac{\sqrt{3}}{2}$.

19.



a) The graphs have the same maximum and minimum values, the same period, and the same domain and range.

b) The graphs have different x- and y-intercepts.

c) A horizontal translation could make them the same graph.

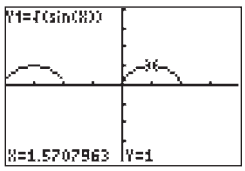
20. 12

21. a) $\frac{2\pi}{3}$ **b)** 12

22. 0.9

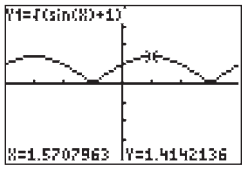
23. a) Example: The graph of $y = \sqrt{\sin x}$ will contain the portions of the graph of $y = \sin x$ that lie on or above the x-axis.

b)



c) Example: The function $y = \sqrt{\sin x + 1}$ is defined for all values of x, while the function $y = \sqrt{\sin x}$ is not.

d)



24. It is sinusoidal and the period is 2π.

C1 Step 5

a) The x-coordinate of each point on the unit circle represents $\cos\theta$. The y-coordinate of each point on the unit circle represents the $\sin\theta$.

b) The y-coordinates of the points on the sine graph are the same as the y-coordinates of the points on the unit circle. The y-coordinates of the points on the cosine graph are the same as the x-coordinates of the points on the unit circle.

C2 The constant is 1. The sum of the squares of the legs of each right triangle is equal to the radius of the unit circle, which is always 1.

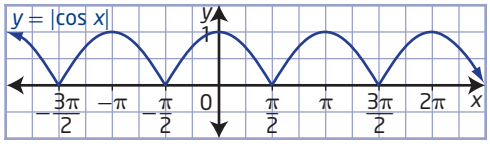
C3 a) Cannot determine because the amplitude is not given.

b) $f(4) = 0$; given in the question.

c) $f(84) = 0$; the period is 40° so it returns to 0 every 40°.

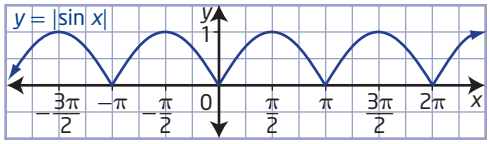
C4 a) Sine and Cosine **b)** Sine and Cosine
c) Sine and Cosine **d)** Sine and Cosine
e) Sine **f)** Cosine **g)** Cosine **h)** Sine
i) Cosine **j)** Sine **k)** Cosine **l)** Sine
m) Sine **n)** Cosine

C5 a)



The parts of the graph below the x-axis have been reflected across the x-axis.

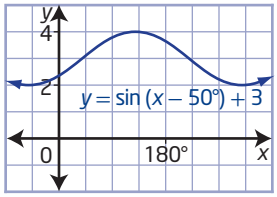
b)



The parts of the graph below the x-axis have been reflected across the x-axis.

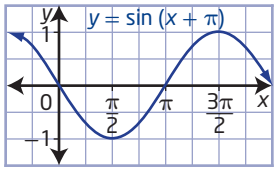
5.2 Transformations of Sinusoidal Functions, pages 250 to 255

1. a)



The phase shift is 50° right. The vertical displacement is 3 units up.

b)



The phase shift is π units left. There is no vertical displacement.