Check Your Understanding

Practise

1. Determine the phase shift and the vertical displacement with respect to $y = \sin x$ for each function. Sketch a graph of each function.

a) $y = \sin (x - 50°) + 3$

b) $y = \sin (x + \pi)$

c) $y = \sin \left(x + \frac{2\pi}{3}\right) + 5$

d) $y = 2 \sin (x + 50°) - 10$

e) $y = -3 \sin (6x + 30°) - 3$

f) $y = 3 \sin \frac{1}{2}\left(x - \frac{\pi}{4}\right) - 10$

2. Determine the phase shift and the vertical displacement with respect to $y = \cos x$ for each function. Sketch a graph of each function.

a) $y = \cos (x - 30°) + 12$

b) $y = \cos \left(x - \frac{\pi}{3}\right)$

c) $y = \cos \left(x + \frac{5\pi}{6}\right) + 16$

d) $y = 4 \cos (x + 15°) + 3$

e) $y = 4 \cos (x - \pi) + 4$

f) $y = 3 \cos \left(2x - \frac{\pi}{6}\right) + 7$

3. a) Determine the range of each function.

i) $y = 3 \cos \left(x - \frac{\pi}{2}\right) + 5$

ii) $y = -2 \sin (x + \pi) - 3$

iii) $y = 1.5 \sin x + 4$

iv) $y = \frac{2}{3} \cos (x + 50°) + \frac{3}{4}$

b) Describe how to determine the range when given a function of the form $y = a \cos b(x - c) + d$ or $y = a \sin b(x - c) + d$.

4. Match each function with its description in the table.

a) $y = -2 \cos 2(x + 4) - 1$

b) $y = 2 \sin 2(x - 4) - 1$

c) $y = 2 \sin (2x - 4) - 1$

d) $y = 3 \sin (3x - 9) - 1$

e) $y = 3 \sin (3x + \pi) - 1$

	Amplitude	Period	Phase Shift	Vertical Displacement
A	3	$\frac{2\pi}{3}$	3 right	1 down
B	2	π	2 right	1 down
C	2	π	4 right	1 down
D	2	π	4 left	1 down
E	3	$\frac{2\pi}{3}$	$\frac{\pi}{3}$ left	1 down

5. Match each function with its graph.

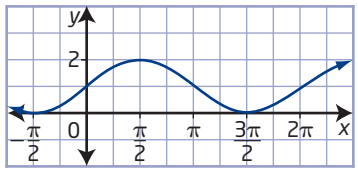
a) $y = \sin \left(x - \frac{\pi}{4}\right)$

b) $y = \sin \left(x + \frac{\pi}{4}\right)$

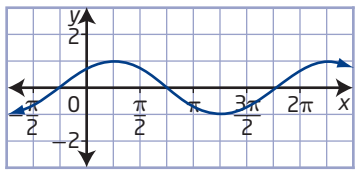
c) $y = \sin x - 1$

d) $y = \sin x + 1$

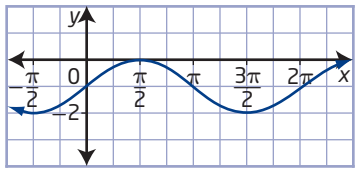
A



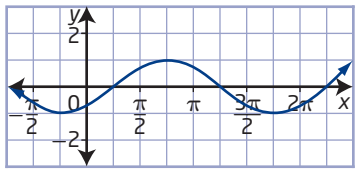
B



C



D



Apply

6. Write the equation of the sine function in the form $y = a \sin b(x - c) + d$ given its characteristics.

a) amplitude 4, period π, phase shift $\frac{\pi}{2}$ to the right, vertical displacement 6 units down

b) amplitude 0.5, period 4π, phase shift $\frac{\pi}{6}$ to the left, vertical displacement 1 unit up

c) amplitude $\frac{3}{4}$, period 720°, no phase shift, vertical displacement 5 units down

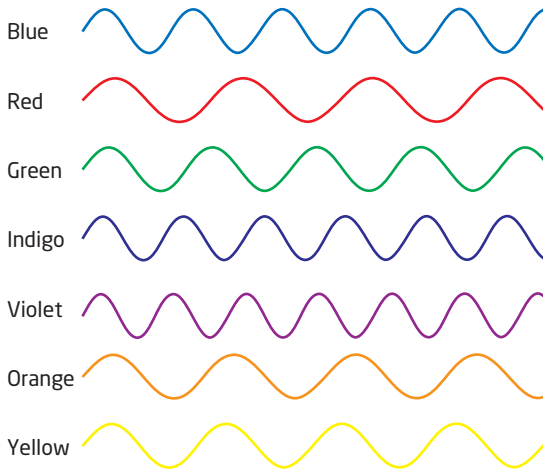
7. The graph of $y = \cos x$ is transformed as described. Determine the values of the parameters a, b, c, and d for the transformed function. Write the equation for the transformed function in the form $y = a \cos b(x - c) + d$.

a) vertical stretch by a factor of 3 about the x-axis, horizontal stretch by a factor of 2 about the y-axis, translated 2 units to the left and 3 units up

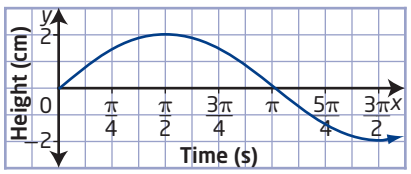
b) vertical stretch by a factor of $\frac{1}{2}$ about the x-axis, horizontal stretch by a factor of $\frac{1}{4}$ about the y-axis, translated 3 units to the right and 5 units down

c) vertical stretch by a factor of $\frac{3}{2}$ about the x-axis, horizontal stretch by a factor of 3 about the y-axis, reflected in the x-axis, translated $\frac{\pi}{4}$ units to the right and 1 unit down

8. When white light shines through a prism, the white light is broken into the colours of the visible light spectrum. Each colour corresponds to a different wavelength of the electromagnetic spectrum. Arrange the colours, in order from greatest to smallest period.



9. The piston engine is the most commonly used engine in the world. The height of the piston over time can be modelled by a sine curve. Given the equation for a sine curve, $y = a \sin b(x - c) + d$, which parameter(s) would be affected as the piston moves faster?

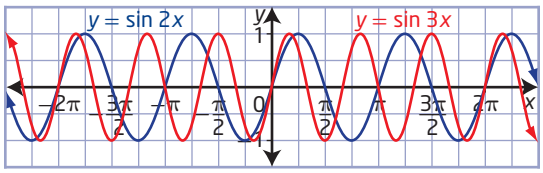


12. a) $B\left(\frac{\pi}{4}, 3\right)$, $C\left(\frac{\pi}{2}, 0\right)$, $D\left(\frac{3\pi}{4}, -3\right)$, $E(\pi, 0)$

b) $C\left(\frac{\pi}{2}, 0\right)$, $D(\pi, -2)$, $E\left(\frac{3\pi}{2}, 0\right)$, $F(2\pi, 2)$

c) $B(-3\pi, 1)$, $C(-2\pi, 0)$, $D(-\pi, -1)$, $E(0, 0)$

13.



The amplitude, maximum, minimum, y-intercepts, domain, and range are the same for both graphs. The period and x-intercepts are different.

14. a) Amplitude is 5; period is $\frac{4\pi}{3}$.

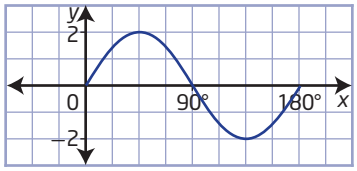
b) Amplitude is 4; Period is $\frac{2\pi}{3}$.

15. a) Amplitude is 20 mm Hg; Period is 0.8 s.

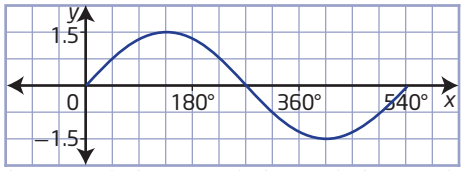
b) 75 bpm

16. Answers may vary.

17. a)



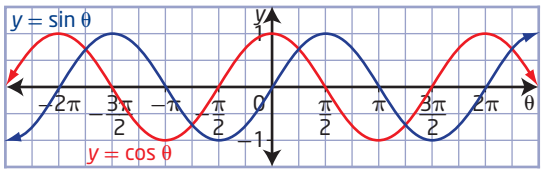
b)



18. a) $\left(-\frac{7\pi}{4}, \frac{\sqrt{2}}{2}\right)$, $\left(-\frac{5\pi}{4}, \frac{\sqrt{2}}{2}\right)$, $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$, $\left(\frac{9\pi}{4}, \frac{\sqrt{2}}{2}\right)$;
Find the points of intersection of $y = \sin\theta$ and $y = \frac{\sqrt{2}}{2}$.

b) $\left(-\frac{11\pi}{6}, \frac{\sqrt{3}}{2}\right)$, $\left(-\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right)$, $\left(\frac{11\pi}{6}, \frac{\sqrt{3}}{2}\right)$, $\left(\frac{13\pi}{6}, \frac{\sqrt{3}}{2}\right)$;
Find the points of intersection of $y = \cos\theta$ and $y = \frac{\sqrt{3}}{2}$.

19.



a) The graphs have the same maximum and minimum values, the same period, and the same domain and range.

b) The graphs have different x- and y-intercepts.

c) A horizontal translation could make them the same graph.

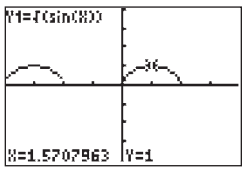
20. 12

21. a) $\frac{2\pi}{3}$ **b)** 12

22. 0.9

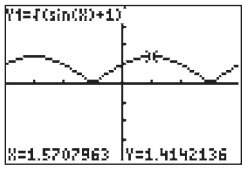
23. a) Example: The graph of $y = \sqrt{\sin x}$ will contain the portions of the graph of $y = \sin x$ that lie on or above the x-axis.

b)



c) Example: The function $y = \sqrt{\sin x + 1}$ is defined for all values of x, while the function $y = \sqrt{\sin x}$ is not.

d)



24. It is sinusoidal and the period is 2π.

C1 Step 5

a) The x-coordinate of each point on the unit circle represents $\cos\theta$. The y-coordinate of each point on the unit circle represents the $\sin\theta$.

b) The y-coordinates of the points on the sine graph are the same as the y-coordinates of the points on the unit circle. The y-coordinates of the points on the cosine graph are the same as the x-coordinates of the points on the unit circle.

C2 The constant is 1. The sum of the squares of the legs of each right triangle is equal to the radius of the unit circle, which is always 1.

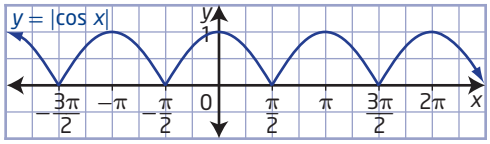
C3 a) Cannot determine because the amplitude is not given.

b) $f(4) = 0$; given in the question.

c) $f(84) = 0$; the period is 40° so it returns to 0 every 40°.

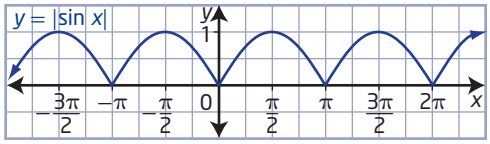
C4 a) Sine and Cosine **b)** Sine and Cosine
c) Sine and Cosine **d)** Sine and Cosine
e) Sine **f)** Cosine **g)** Cosine **h)** Sine
i) Cosine **j)** Sine **k)** Cosine **l)** Sine
m) Sine **n)** Cosine

C5 a)



The parts of the graph below the x-axis have been reflected across the x-axis.

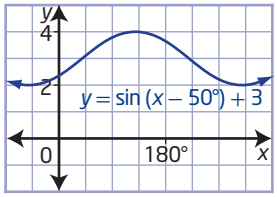
b)



The parts of the graph below the x-axis have been reflected across the x-axis.

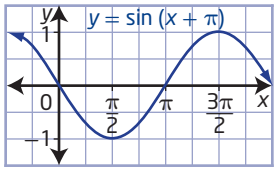
5.2 Transformations of Sinusoidal Functions, pages 250 to 255

1. a)

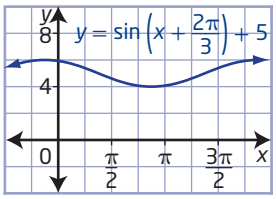


The phase shift is 50° right. The vertical displacement is 3 units up.

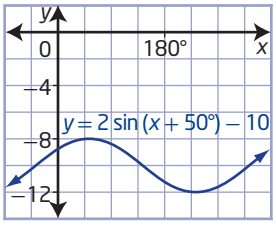
b)



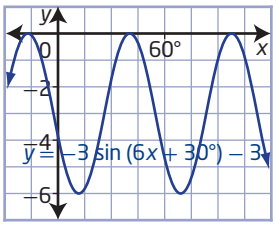
The phase shift is π units left. There is no vertical displacement.

c) 

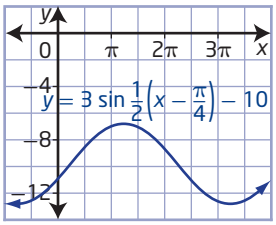
The phase shift is $\frac{2\pi}{3}$ units left. The vertical displacement is 5 units up.

d) 

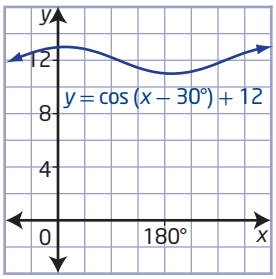
The phase shift is 50° left. The vertical displacement is 10 units down.

e) 

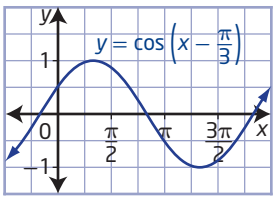
The phase shift is 5° left. The vertical displacement is 3 units down.

f) 

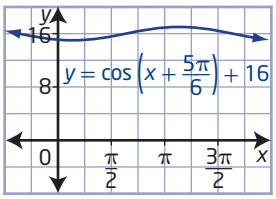
The phase shift is $\frac{\pi}{4}$ units right. The vertical displacement is 10 units down.

2. a) 

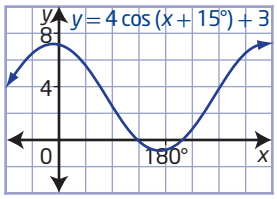
The phase shift is 30° right. The vertical displacement is 12 units up.

b) 

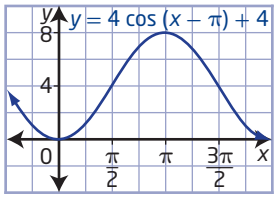
The phase shift is $\frac{\pi}{3}$ units right. There is no vertical displacement.

c) 

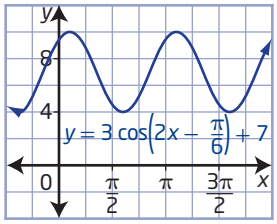
The phase shift is $\frac{5\pi}{6}$ units left. The vertical displacement is 16 units up.

d) 

The phase shift is 15° left. The vertical displacement is 3 units up.

e) 

The phase shift π units right. The vertical displacement is 4 units up.

f) 

The phase shift is $\frac{\pi}{12}$ units right. The vertical displacement is 7 units up.

3. a) i) $\{y \mid 2 \le y \le 8, y \in \mathrm{R}\}$
ii) $\{y \mid -5 \le y \le -1, y \in \mathrm{R}\}$
iii) $\{y \mid 2.5 \le y \le 5.5, y \in \mathrm{R}\}$
iv) $\left\{y \mid \frac{1}{12} \le y \le \frac{17}{12}, y \in \mathrm{R}\right\}$

b) Take the vertical displacement and add and subtract the amplitude to it. The region in between these points is the range.

4. a) D b) C c) B d) A e) E

5. a) D b) B c) C d) A

6. a) $y = 4\sin 2\left(x - \frac{\pi}{2}\right) - 6$

b) $y = 0.5\sin\frac{1}{2}\left(x + \frac{\pi}{6}\right) + 1$

c) $y = \frac{3}{4}\sin\frac{1}{2}x - 5$

7. a) $a = 3, b = \frac{1}{2}, c = -2, d = 3; y = 3\cos\frac{1}{2}(x + 2) + 3$

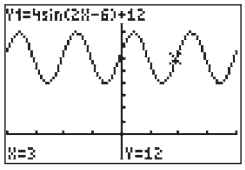
b) $a = \frac{1}{2}, b = 4, c = 3, d = -5;$
$y = \frac{1}{2}\cos 4(x - 3) - 5$

c) $a = -\frac{3}{2}, b = \frac{1}{3}, c = \frac{\pi}{4}, d = -1;$
$y = -\frac{3}{2}\cos\frac{1}{3}\left(x - \frac{\pi}{4}\right) - 1$

8. red, orange, yellow, green, blue, indigo, violet

9. b

10. a) Stewart is correct. He remembered to factor the expression in brackets first.

b) 

11. a) $\{y \mid -1 \le y \le 5, y \in \mathrm{R}\}$ b) $\{y \mid -6 \le y \le 0, y \in \mathrm{R}\}$
c) $\{y \mid -13 \le y \le -7, y \in \mathrm{R}\}$
d) $\{y \mid 5 \le y \le 11, y \in \mathrm{R}\}$