Key Ideas

- Angles can be measured using different units, including degrees and radians.
- An angle measured in one unit can be converted to the other unit using the relationships 1 full rotation $= 360° = 2\pi$.
- An angle in standard position has its vertex at the origin and its initial arm along the positive x-axis.
- Angles that are coterminal have the same initial arm and the same terminal arm.
- An angle θ has an infinite number of angles that are coterminal expressed by $\theta \pm (360°)n$, $n \in \mathrm{N}$, in degrees, or $\theta \pm 2\pi n$, $n \in \mathrm{N}$, in radians.
- The formula $a = \theta r$, where a is the arc length; θ is the central angle, in radians; and r is the length of the radius, can be used to determine any of the variables given the other two, as long as a and r are in the same units.

Check Your Understanding

Practise

1. For each angle, indicate whether the direction of rotation is clockwise or counterclockwise.

a) -4π **b)** $750°$

c) $-38.7°$ **d)** 1

2. Convert each degree measure to radians. Write your answers as exact values. Sketch the angle and label it in degrees and in radians.

a) $30°$ **b)** $45°$

c) $-330°$ **d)** $520°$

e) $90°$ **f)** $21°$

3. Convert each degree measure to radians. Express your answers as exact values and as approximate measures, to the nearest hundredth of a radian.

a) $60°$ **b)** $150°$

c) $-270°$ **d)** $72°$

e) $-14.8°$ **f)** $540°$

4. Convert each radian measure to degrees. Express your answers as exact values and as approximate measures, to the nearest tenth of a degree, if necessary.

a) $\frac{\pi}{6}$ **b)** $\frac{2\pi}{3}$

c) $-\frac{3\pi}{8}$ **d)** $-\frac{5\pi}{2}$

e) 1 **f)** 2.75

5. Convert each radian measure to degrees. Express your answers as exact values and as approximate measures, to the nearest thousandth.

a) $\frac{2\pi}{7}$ **b)** $\frac{7\pi}{13}$

c) $\frac{2}{3}$ **d)** 3.66

e) -6.14 **f)** -20

6. Sketch each angle in standard position. In which quadrant does each angle terminate?

a) 1 **b)** $-225°$

c) $\frac{17\pi}{6}$ **d)** $650°$

e) $-\frac{2\pi}{3}$ **f)** $-42°$

7. Determine one positive and one negative angle coterminal with each angle.

a) $72°$ **b)** $\frac{3\pi}{4}$

c) $-120°$ **d)** $\frac{11\pi}{2}$

e) $-205°$ **f)** 7.8

8. Determine whether the angles in each pair are coterminal. For one pair of angles, explain how you know.

a) $\frac{5\pi}{6}, \frac{17\pi}{6}$ **b)** $\frac{5\pi}{2}, -\frac{9\pi}{2}$

c) $410°, -410°$ **d)** $227°, -493°$

9. Write an expression for all of the angles coterminal with each angle. Indicate what your variable represents.

a) $135°$ **b)** $-\frac{\pi}{2}$

c) $-200°$ **d)** 10

10. Draw and label an angle in standard position with negative measure. Then, determine an angle with positive measure that is coterminal with your original angle. Show how to use a general expression for coterminal angles to find the second angle.

11. For each angle, determine all angles that are coterminal in the given domain.

a) $65°, 0° \le \theta < 720°$

b) $-40°, -180° \le \theta < 360°$

c) $-40°, -720° \le \theta < 720°$

d) $\frac{3\pi}{4}, -2\pi \le \theta < 2\pi$

e) $-\frac{11\pi}{6}, -4\pi \le \theta < 4\pi$

f) $\frac{7\pi}{3}, -2\pi \le \theta < 4\pi$

g) $2.4, -2\pi \le \theta < 2\pi$

h) $-7.2, -4\pi \le \theta < 2\pi$

12. Determine the arc length subtended by each central angle. Give answers to the nearest hundredth of a unit.

a) radius 9.5 cm, central angle 1.4

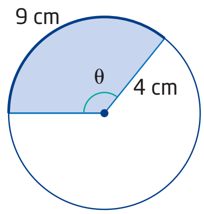
b) radius 1.37 m, central angle 3.5

c) radius 7 cm, central angle 130°

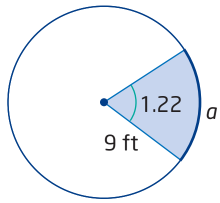
d) radius 6.25 in., central angle 282°

13. Use the information in each diagram to determine the value of the variable. Give your answers to the nearest hundredth of a unit.

a)



b)



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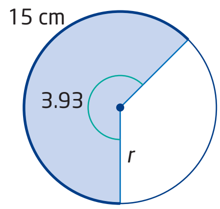
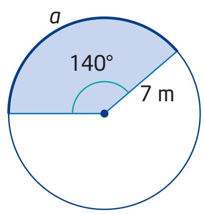


d)



Apply

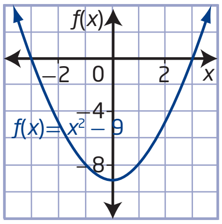
14. A rotating water sprinkler makes one revolution every 15 s. The water reaches a distance of 5 m from the sprinkler.

a) What is the arc length of the sector watered when the sprinkler rotates through $\frac{5\pi}{3}$? Give your answer as both an exact value and an approximate measure, to the nearest hundredth.

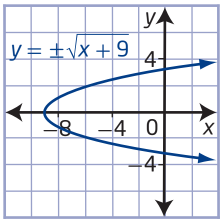
b) Show how you could find the area of the sector watered in part a).

c) What angle does the sprinkler rotate through in 2 min? Express your answer in radians and degrees.

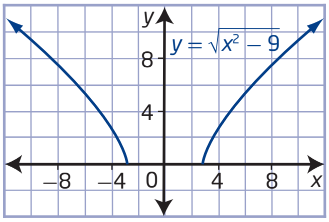
13. a)



b) $y = \pm\sqrt{x + 9}$



c) $y = \sqrt{x^2 - 9}$



d) for part a): domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq -9, y \in \mathrm{R}\}$; for part b): domain $\{x \mid x \geq -9, x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$; for part c): domain $\{x \mid x \leq -3 \text{ or } x \geq 3, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

14. Quadrant II: reflection in the y-axis, $y = f(-x)$; quadrant III: reflection in the y-axis and then the x-axis, $y = -f(-x)$; quadrant IV: reflection in the x-axis, $y = -f(x)$

15. a) Mary should have subtracted 4 from both sides in step 1. She also incorrectly squared the expression on the right side in step 2. The correct solution follows:

$2x = \sqrt{x + 1} + 4$

Step 1: $(2x - 4)^2 = (\sqrt{x + 1})^2$

Step 2: $4x^2 - 16x + 16 = x + 1$

Step 3: $4x^2 - 17x + 15 = 0$

Step 4: $(4x - 5)(x - 3) = 0$

Step 5: $4x - 5 = 0$ or $x - 3 = 0$

Step 6: $x = \frac{5}{4}$ $\quad x = 3$

Step 7: A check determines that $x = 3$ is the solution.

b) Yes, the point of intersection of the two graphs will yield the possible solution, $x = 3$.

16. $c = -3$; $P(x) = (x + 3)(x + 2)(x - 1)^2$

17. a) $\pm 1, \pm 2, \pm 3, \pm 6$

b) $P(x) = (x - 3)(x + 2)(x + 1)$

c) x-intercepts: -2, -1 and 3; y-intercept: -6

d) $-2 \leq x \leq -1$ and $x \geq 3$

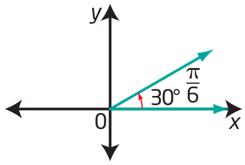
Chapter 4 Trigonometry and the Unit Circle

4.1 Angles and Angle Measure, pages 175 to 179

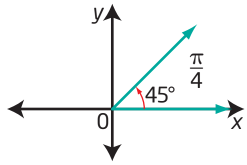
1. a) clockwise **b)** counterclockwise

c) clockwise **d)** counterclockwise

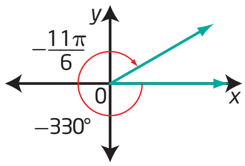
2. a) $30° = \frac{\pi}{6}$



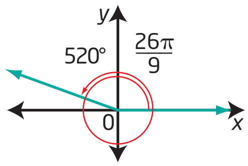
b) $45° = \frac{\pi}{4}$



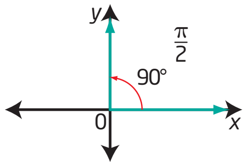
c) $-330° = -\frac{11\pi}{6}$



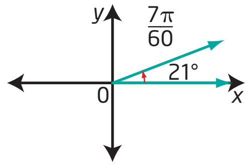
d) $520° = \frac{26\pi}{9}$



e) $90° = \frac{\pi}{2}$



f) $21° = \frac{7\pi}{60}$



3. a) $\frac{\pi}{3}$ or 1.05 **b)** $\frac{5\pi}{6}$ or 2.62

c) $-\frac{3\pi}{2}$ or -4.71 **d)** $\frac{2\pi}{5}$ or 1.26

e) $-\frac{37\pi}{450}$ or -0.26 **f)** 3π or 9.42

4. a) 30° **b)** 120°

c) –67.5° **d)** –450°

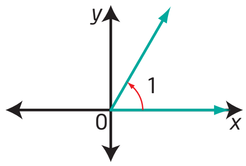
e) $\frac{180°}{\pi}$ or 57.3° **f)** $\frac{495°}{\pi}$ or 157.6°

5. a) $\frac{360°}{7}$ or 51.429° **b)** $\frac{1260°}{13}$ or 96.923°

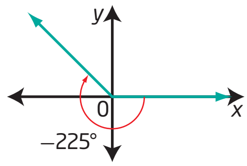
c) $\frac{120°}{\pi}$ or 38.197° **d)** $\frac{3294°}{5\pi}$ or 209.703°

e) $\frac{-1105.2°}{\pi}$ or –351.796° **f)** $\frac{-3600°}{\pi}$ or –1145.916°

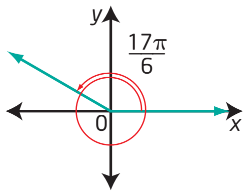
6. a) quadrant I



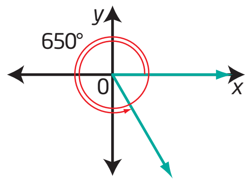
b) quadrant II



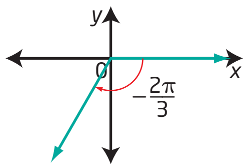
c) quadrant II



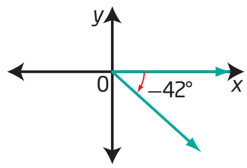
d) quadrant IV



e) quadrant III



f) quadrant IV



7. Examples:

a) 432°, −288° **b)** $\frac{11\pi}{4}, -\frac{5\pi}{4}$

c) 240°, −480° **d)** $\frac{7\pi}{2}, -\frac{\pi}{2}$

e) 155°, −565° **f)** 1.5, −4.8

8. a) coterminal, $\frac{17\pi}{6} = \frac{5\pi}{6} + \frac{12\pi}{6} = \frac{5\pi}{6} + 2\pi$

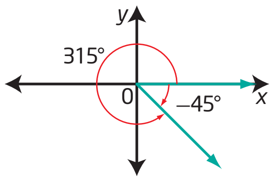
b) not coterminal **c)** not coterminal

d) coterminal, $-493° = 227° - 2(360°)$

9. a) $135° \pm (360°)n,\ n \in \mathrm{N}$ **b)** $-\frac{\pi}{2} \pm 2\pi n,\ n \in \mathrm{N}$

c) $-200° \pm (360°)n,\ n \in \mathrm{N}$ **d)** $10 \pm 2\pi n,\ n \in \mathrm{N}$

10. Example:



$-45° + 360° = 315°,\ -45° \pm (360°)n,\ n \in \mathrm{N}$

11. a) 425° **b)** 320°

c) −400°, 320°, 680° **d)** $-\frac{5\pi}{4}$

e) $-\frac{23\pi}{6}, \frac{\pi}{6}, \frac{13\pi}{6}$ **f)** $-\frac{5\pi}{3}, \frac{\pi}{3}$

g) −3.9 **h)** −0.9, 5.4

12. a) 13.30 cm **b)** 4.80 m

c) 15.88 cm **d)** 30.76 in.

13. a) 2.25 radians **b)** 10.98 ft

c) 3.82 cm **d)** 17.10 m

14. a) $\frac{25\pi}{3}$ or 26.18 m

b) $\frac{A_{\text{sector}}}{A_{\text{circle}}} = \frac{\text{sector angle}}{2\pi}$

$A_{\text{sector}} = \frac{\pi r^2\left(\frac{5\pi}{3}\right)}{2\pi}$

$A_{\text{sector}} = \frac{5\pi(5)^2}{6}$

$A_{\text{sector}} = \frac{125\pi}{6}$

The area watered is approximately 65.45 m².

c) 16π radians or 2880°

15. a) Examples: $\frac{\pi}{12}$ radians/h, 1 revolution per day, 15°/h

b) $\frac{100\pi}{3}$ or 104.72 radians/s

c) 54 000°/min

16. a) 2.36 **b)** 135.3°

17.

	Revolutions	Degrees	Radians
a)	1 rev	360°	2π
b)	0.75 rev	270°	$\frac{3\pi}{2}$ or 4.7
c)	0.4 rev	150°	$\frac{5\pi}{6}$
d)	−0.3 rev	−97.4°	−1.7
e)	−0.1 rev	−40°	$-\frac{2\pi}{9}$ or −0.7
f)	0.7 rev	252°	$\frac{7\pi}{5}$ or 4.4
g)	−3.25 rev	−1170°	$-\frac{13\pi}{2}$ or −20.4
h)	$\frac{23}{18}$ or 1.3 rev	460°	$\frac{23\pi}{9}$ or 8.0
i)	$-\frac{3}{16}$ or −0.2 rev	−67.5°	$-\frac{3\pi}{8}$

18. Jasmine is correct. Joran's answer includes the solution when $k = 0$, which is the reference angle 78°.

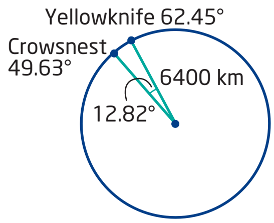
19. a) 55.6 grad

b) Use a proportion: $\frac{\text{gradians}}{\text{degrees}} = \frac{400 \text{ grad}}{360°}$.

So, measure in gradians $= \frac{10(\text{number of degrees})}{9}$.

c) The gradian was developed to express a right angle as a metric measure. A right angle is equivalent to 100 grads.

20. a)



b) 1432.01 km

c) Example: Bowden (51.93° N, 114.03° W) and Airdrie (51.29° N, 114.01° W) are 71.49 km apart.

21. a) 2221.4 m/min **b)** 7404.7 radians/min

22. 8.5 km/h

23. 66 705.05 mph

24. a) $69.375° = 69° + 0.375(60')$

$= 69°\ 22.5'$

$= 69°\ 22'\ 30''$

b) **i)** 40° 52′ 30″ **ii)** 100° 7′ 33.6″

iii) 14° 33′ 54″ **iv)** 80° 23′ 6″

25. a) $69°\ 22'\ 30'' = 69°22.5'$

$= 69° + \left(\frac{22.5}{60}\right)^{\circ}$

$= 69.375°$

b) **i)** 45.508° **ii)** 72.263°

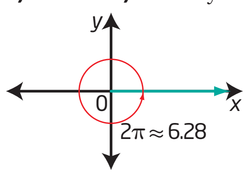
iii) 105.671° **iv)** 28.167°

26. $A_{\text{segment}} = \frac{1}{2}r^2(\theta - \sin\theta)$

27. a) 120° **b)** 65° **c)** Examples: 3:00 and 9:00

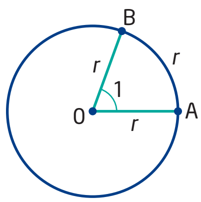
d) 2 **e)** shortly after 4:05

C1



π is 180° and 2π is 360°. 2(3.14) = 6.282 which is more than 6. Therefore, 6 radians must be less than 360°.

C2



1° is a very small angle, it is $\frac{1}{360}$ of one rotation. One radian is much larger than 1°; 1 radian is the angle whose arc is the same as the radius, it is nearly $\frac{1}{6}$ of one rotation.

C3 a) $40°;\ 140° \pm (360°)n,\ n \in \mathrm{N}$

b) $0.72;\ 0.72 \pm 2\pi n,\ n \in \mathrm{N}$

C4 a)