Key Ideas

- Points that are on the intersection of the terminal arm of an angle θ in standard position and the unit circle can be defined using trigonometric ratios.

  $P(\theta) = (\cos \theta, \sin \theta)$

- Each primary trigonometric ratio—sine, cosine, and tangent—has a reciprocal trigonometric ratio. The reciprocals are cosecant, secant, and cotangent, respectively.

  $\csc \theta = \dfrac{1}{\sin \theta}$ $\sec \theta = \dfrac{1}{\cos \theta}$ $\cot \theta = \dfrac{1}{\tan \theta}$

  If $\sin \theta = \frac{2}{3}$, then $\csc \theta = \frac{3}{2}$, and vice versa.

- You can determine the trigonometric ratios for any angle in standard position using the coordinates of the point where the terminal arm intersects the unit circle.
- Exact values of trigonometric rations for special angles such as 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, and $\frac{\pi}{2}$ and their multiples may be determined using the coordinates of points on the unit circle.
- You can determine approximate values for trigonometric ratios using a calculator in the appropriate mode: radians or degrees.
- You can use a scientific or graphing calculator to determine an angle measure given the value of a trigonometric ratio. Then, use your knowledge of reference angles, coterminal angles, and signs of ratios in each quadrant to determine other possible angle measures. Unless the domain is restricted, there are an infinite number of answers.
- Determine the trigonometric ratios for an angle θ in standard position from the coordinates of a point on the terminal arm of θ and right triangle definitions of the trigonometric ratios.

Check Your Understanding

Practise

1. What is the exact value for each trigonometric ratio?

a) $\sin 45°$ **b)** $\tan 30°$

c) $\cos \frac{3\pi}{4}$ **d)** $\cot \frac{7\pi}{6}$

e) $\csc 210°$ **f)** $\sec (-240°)$

g) $\tan \frac{3\pi}{2}$ **h)** $\sec \pi$

i) $\cot (-120°)$ **j)** $\cos 390°$

k) $\sin \frac{5\pi}{3}$ **l)** $\csc 495°$

2. Determine the approximate value for each trigonometric ratio. Give answers to two decimal places.

a) $\cos 47°$ **b)** $\cot 160°$

c) $\sec 15°$ **d)** $\csc 4.71$

e) $\sin 5$ **f)** $\tan 0.94$

g) $\sin \frac{5\pi}{7}$ **h)** $\tan 6.9$

i) $\cos 302°$ **j)** $\sin \left(-\frac{11\pi}{19}\right)$

k) $\cot 6$ **l)** $\sec (-270°)$

3. If θ is an angle in standard position with the following conditions, in which quadrants may θ terminate?

a) $\cos \theta > 0$

b) $\tan \theta < 0$

c) $\sin \theta < 0$

d) $\sin \theta > 0$ and $\cot \theta < 0$

e) $\cos \theta < 0$ and $\csc \theta > 0$

f) $\sec \theta > 0$ and $\tan \theta > 0$

4. Express the given quantity using the same trigonometric ratio and its reference angle. For example, $\cos 110° = -\cos 70°$. For angle measures in radians, give exact answers. For example, $\cos 3 = -\cos (\pi - 3)$.

a) $\sin 250°$ **b)** $\tan 290°$

c) $\sec 135°$ **d)** $\cos 4$

e) $\csc 3$ **f)** $\cot 4.95$

5. For each point, sketch two coterminal angles in standard position whose terminal arm contains the point. Give one positive and one negative angle, in radians, where neither angle exceeds one full rotation.

a) $(3, 5)$ **b)** $(-2, -1)$

c) $(-3, 2)$ **d)** $(5, -2)$

6. Indicate whether each trigonometric ratio is positive or negative. Do not use a calculator.

a) $\cos 300°$ **b)** $\sin 4$

c) $\cot 156°$ **d)** $\csc (-235°)$

e) $\tan \frac{13\pi}{6}$ **f)** $\sec \frac{17\pi}{3}$

7. Determine each value. Explain what the answer means.

a) $\sin^{-1} 0.2$ **b)** $\tan^{-1} 7$

c) $\sec 450°$ **d)** $\cot (-180°)$

8. The point $P(\theta) = \left(\frac{3}{5}, y\right)$ lies on the terminal arm of an angle θ in standard position and on the unit circle. $P(\theta)$ is in quadrant IV.

a) Determine y.

b) What is the value of $\tan \theta$?

c) What is the value of $\csc \theta$?

Apply

9. Determine the exact value of each expression.

a) $\cos 60° + \sin 30°$

b) $(\sec 45°)^2$

c) $\left(\cos \frac{5\pi}{3}\right)\left(\sec \frac{5\pi}{3}\right)$

d) $(\tan 60°)^2 - (\sec 60°)^2$

e) $\left(\cos \frac{7\pi}{4}\right)^2 + \left(\sin \frac{7\pi}{4}\right)^2$

f) $\left(\cot \frac{5\pi}{6}\right)^2$

10. Determine the exact measure of all angles that satisfy the following. Draw a diagram for each.

a) $\sin \theta = -\frac{1}{2}$ in the domain $0 \leq \theta < 2\pi$

b) $\cot \theta = 1$ in the domain $-\pi \leq \theta < 2\pi$

c) $\sec \theta = 2$ in the domain $-180° \leq \theta < 90°$

d) $(\cos \theta)^2 = 1$ in the domain $-360° \leq \theta < 360°$

11. Determine the approximate measure of all angles that satisfy the following. Give answers to two decimal places. Use diagrams to show the possible answers.

a) $\cos \theta = 0.42$ in the domain $-\pi \leq \theta \leq \pi$

b) $\tan \theta = -4.87$ in the domain $-\frac{\pi}{2} \leq \theta \leq \pi$

c) $\csc \theta = 4.87$ in the domain $-360° \leq \theta < 180°$

d) $\cot \theta = 1.5$ in the domain $-180° \leq \theta < 360°$

12. Determine the exact values of the other five trigonometric ratios under the given conditions.

a) $\sin \theta = \frac{3}{5}, \frac{\pi}{2} < \theta < \pi$

b) $\cos \theta = \frac{-2\sqrt{2}}{3}, -\pi \leq \theta \leq \frac{3\pi}{2}$

c) $\tan \theta = \frac{2}{3}, -360° < \theta < 180°$

d) $\sec \theta = \frac{4\sqrt{3}}{3}, -180° \leq \theta \leq 180°$

b) Compare with a quadratic function. When $y = x^2$ is translated so its vertex moves from (0, 0) to (h, k), its equation becomes $y = (x - h)^2 + k$. So, a reasonable conjecture for the circle centre (0, 0) moving its centre to (h, k) is $(x - h)^2 + (y - k)^2 = r^2$. Test some key points on the circle centre (0, 0) such as $(r, 0)$. When the centre moves to (h, k) the test point moves to $(r + h, k)$. Substitute into the left side of the equation.
$(r + h - h)^2 + (k - k)^2 = r^2 + 0 =$ right side.

C4 a) 21.5% **b)** $\pi : 4$

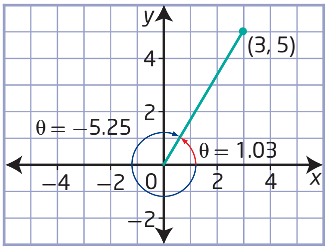
4.3 Trigonometric Ratios, pages 201 to 205

1. a) $\frac{\sqrt{2}}{2}$ **b)** $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$ **c)** $-\frac{\sqrt{2}}{2}$
d) $\sqrt{3}$ **e)** -2 **f)** -2
g) undefined **h)** -1 **i)** $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
j) $\frac{\sqrt{3}}{2}$ **k)** $-\frac{\sqrt{3}}{2}$ **l)** $\sqrt{2}$

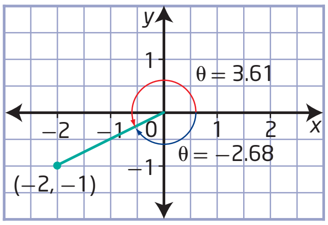
2. a) 0.68 **b)** −2.75 **c)** 1.04
d) −1.00 **e)** −0.96 **f)** 1.37
g) 0.78 **h)** 0.71 **i)** 0.53
j) −0.97 **k)** −3.44 **l)** undefined

3. a) I or IV **b)** II or IV **c)** III or IV
d) II **e)** II **f)** I

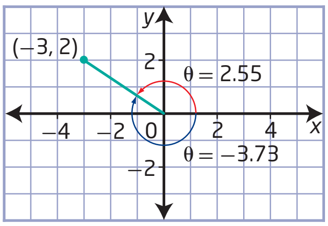
4. a) sin 250° = −sin 70° **b)** tan 290° = −tan 70°
c) sec 135° = −sec 45° **d)** cos 4 = −cos (4 − π)
e) csc 3 = csc (π − 3) **f)** cot 4.95 = cot (4.95 − π)

5. a)


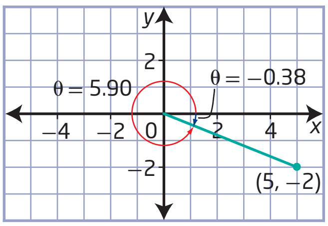
1.03, −5.25

b)


3.61, −2.68

c)


2.55, −3.73

d)


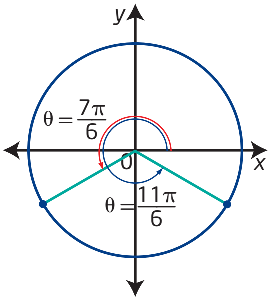
5.90, −0.38

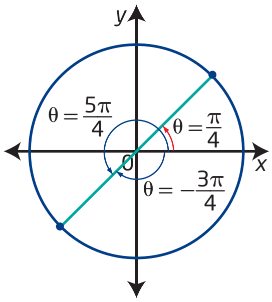
6. a) positive **b)** negative **c)** negative
d) positive **e)** positive **f)** positive

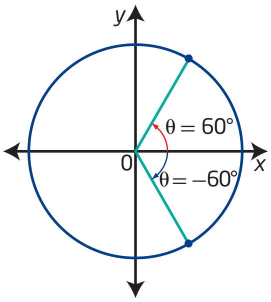
7. a) $\sin^{-1} 0.2014 = 0.2$; an angle of 0.2 radians has a sine ratio of 0.2014
b) $\tan^{-1} 1.429 = 7$; an angle of 7 radians has a tangent ratio of 1.429
c) sec 450° is undefined; an angle of 450° has a secant ratio that is undefined
d) cot (−180°) is undefined; an angle of −180° has a cotangent ratio that is undefined

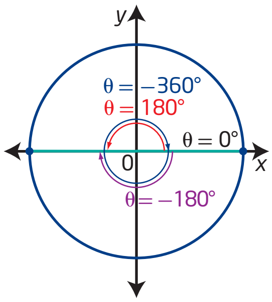
8. a) $-\frac{4}{5}$ **b)** $-\frac{4}{3}$ **c)** −1.25

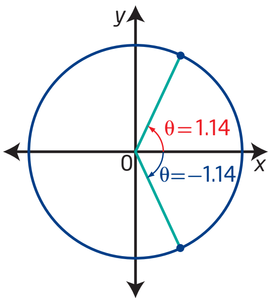
9. a) 1 **b)** 2 **c)** 1
d) −1 **e)** 1 **f)** 3

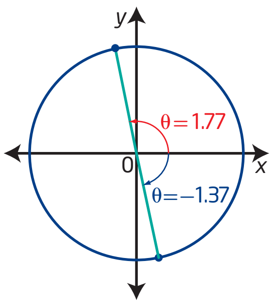
10. a) $\frac{7\pi}{6}, \frac{11\pi}{6}$


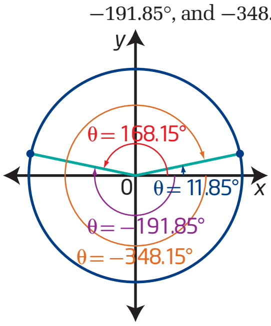
b) $-\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$


c) −60°, 60°


d) −360°, −180°, 0°, 180°


11. a) 1.14 or −1.14


b) −1.37 or 1.77


c) 11.85°, 168.15°, −191.85°, and −348.15°


d) 33.69°, 213.69° and −146.31°